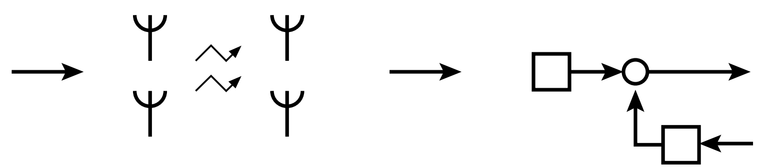
Bit-Interleaved coded modulations for multiple-input multiple output channels



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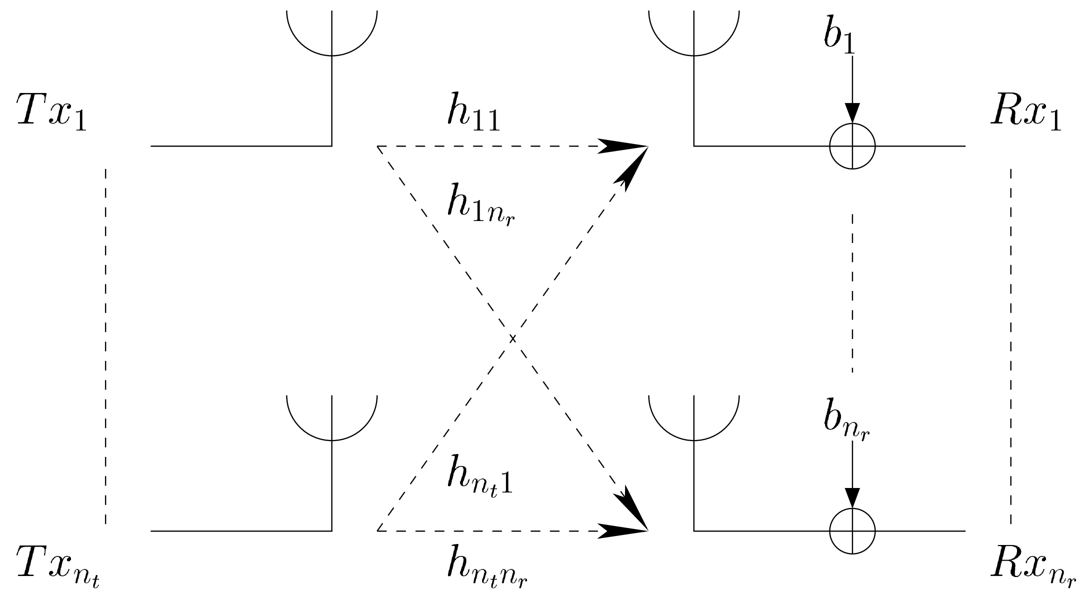
Presentation Outline

- Introduction: the MIMO channel
- Bounds for MIMO systems
- BICM principle and system model
- Signal to APP conversion
- Iterative detection and decoding
- EM estimation of channel parameters
- Results with NRNSC and turbo codes
- MIMO transfer function

Channel model and notations



The MIMO structure contains n_t transmitting antennas (Tx side) and n_r receiving antennas (Rx side).

The received signal vector is given by:

$$\mathbf{y}(k) = H(k)\mathbf{x}(k) + \mathbf{b}(k)$$

where $H = [h_{i,j}]_{i=1\ldots,n_r,j=1,\ldots,n_t}$ is the channel matrix.

Performance of mono-antenna systems

- for an M-QAM over the AWGN channel

$$P_{eb1} \leq \frac{4}{\log_2 M} Q\left(\sqrt{\frac{2E_b}{N_0}\frac{3\log_2 M}{2(M-1)}}\right),$$

where E_b is the mean energy per bit, $N_0/2$ the white additive Gaussian noise spectral density and Q is the error fonction.

- for an M-QAM over the Rayleigh channel

$$P_{eb2} \leq \frac{2}{\log_2 M}\left(\frac{3\log_2 M}{2(M-1)} \times \frac{E_b}{N_0} + 1\right)^{-1} .$$

Performance of multiple-antennas systems

- over the Rayleigh channel

The pairwise error probability $P(U \to V)$, *i.e.* the probability to decode $V = (V_1, \ldots, V_{n_t})$ when $U = (U_1, \ldots, U_{n_t})$ was emitted, is given by:

$$P(U \to V) \leq \frac{1}{2} \left[\frac{1}{1 + \frac{\sum_{i=1}^{n_t} |V_i - U_i|^2}{8N_0}} \right]^{n_r}$$

- over the block Rayleigh channel

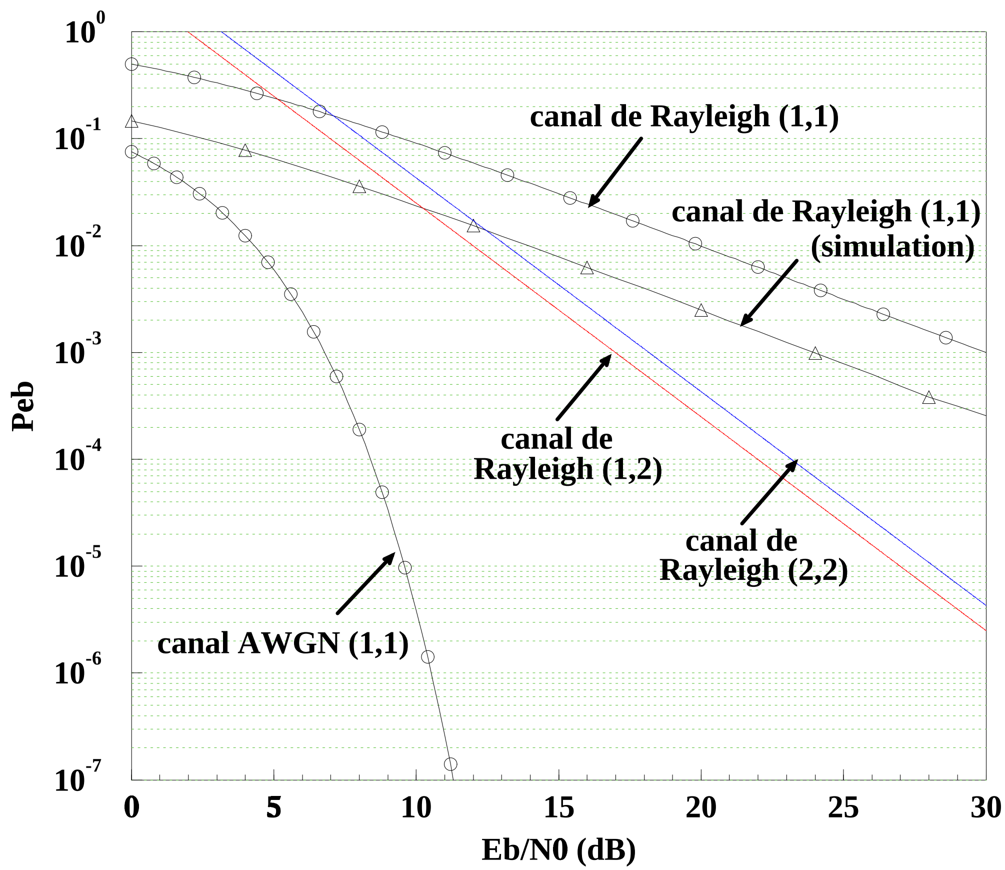
When the Rayleigh channel is constant over ℓ symbols, an heavier derivation can be done, leading to the following expression of the pairwise error probability :

$$P(U \to V) \leq \frac{1}{2} \left(\prod_{i=1}^{r} \frac{\lambda_i}{E_b} \right)^{-n_r} \left(\frac{E_b}{8N_0} \right)^{-rn_r}$$

where the $\lambda_i, i = 1, \ldots, r$ are the eigenvalues of the positive definite Hermitian matrix $A(U,V) = (A_{pq})_{p,q=1,\ldots,n_t}$ where $A_{pq} = \Sigma_{k=1}^{\ell} (V_p^k - U_p^k)(V_q^k - U_q^k)^*$.

☞ diversity gain: rn_r, and coding gain $\left(\Pi_{i=1}^{r} \frac{\lambda_i}{E_s} \right)^{1/n_r}$

Example: bounds on error probability for a QPSK over various channels



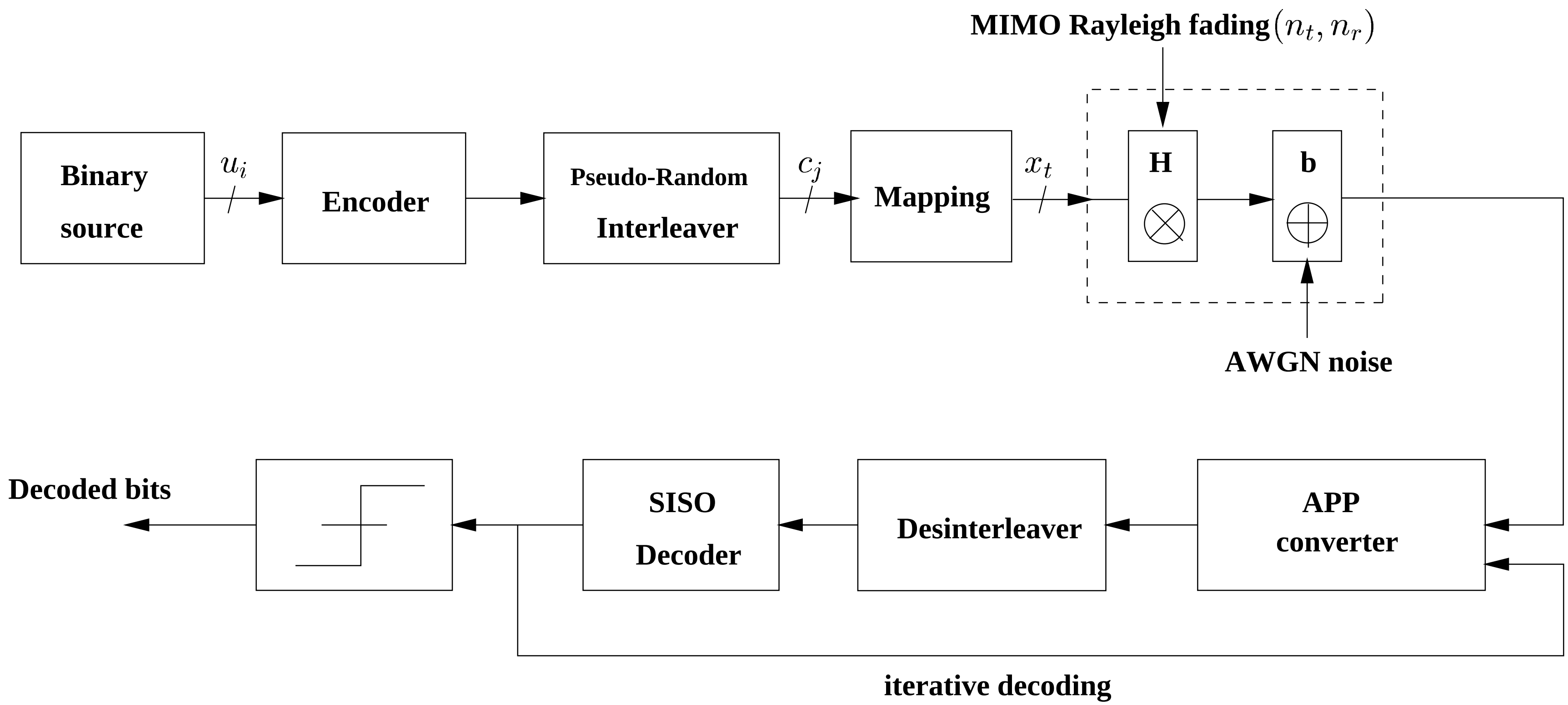
Note the slope variation when n_r goes from 1 to 2 antennas.

Bit Interleaved Coded Modulations principle and historical background

- 1982 and Ungerboeck famous article: modulation and coding should be combined in a single entity for improved performance.
- end 80's-90's: strong interest in mobile-radio channels. The "Ungerboeck paradigm" leads to keep coding combined with modulation.
- 1992 (Zehavi): first glimpses on using interleaving to separate coding and modulation
- 1998 (Caire *et al.*): theoretical approach to BICM, showing how on some channels the separation of demodulation and decoding might be beneficial, provided that the encoder output is interleaved bit-wise and a suitable soft-decision metric is used in the decoder.

☞ For fading channels, the code performance depends strongly on its minimum Hamming distance (the "code diversity"), rather than on the minimum Euclidean distance

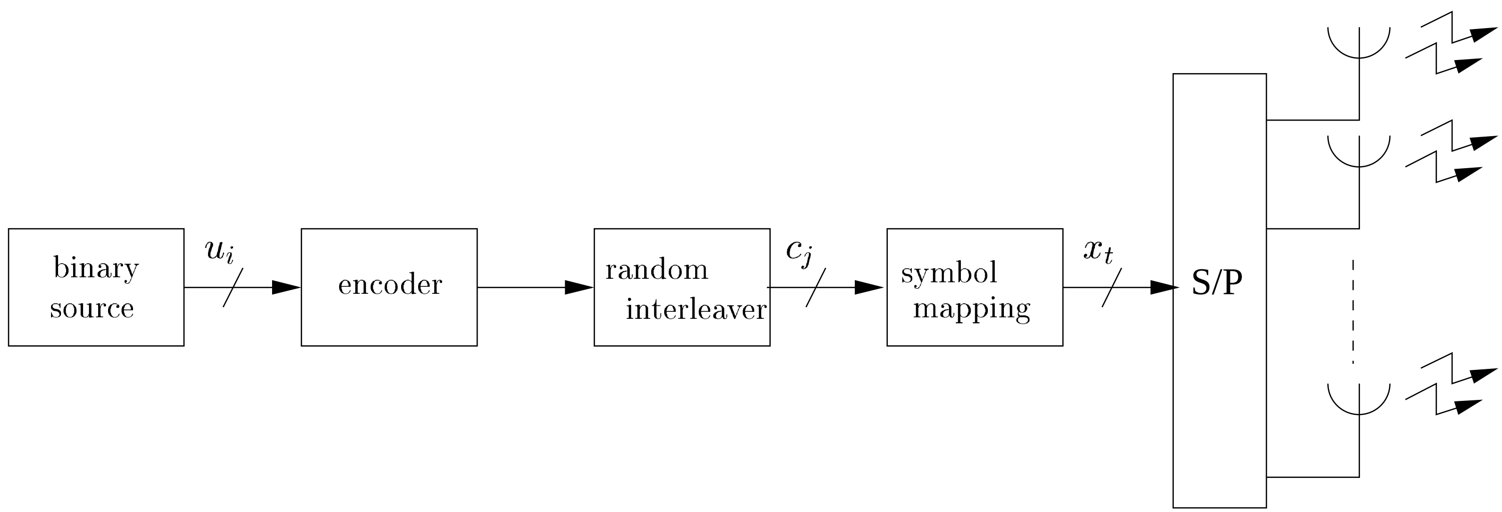
Bit Interleaved Coded Modulations system model



The considered BICM allows to separate the coding operation from the large spectral efficiency modulation one.

Multiple Antennas Bitwise Transmitter

The emitter structure is as follows:



The noise **b** over the channel is assumed to be a additive white Gaussian. The received signal vector is $\mathbf{y}(k) = H(k) \cdot \mathbf{x}(k) + \mathbf{n}(k)$ where $H(k) = [h_{i,j}(k)]_{i=1..n_r, j=1..n_t}$ is the channel matrix.

The symbols x_j belong to a PSK or a QAM constellation of size $M = 2^m$.
The fading coefficients $h_{i,j}(k) \in \mathbb{C}$ are Gaussian and mutually independent.

Signal to APP Conversion

A posteriori probability at the channel output:

$$APP(c_j) \ = \ p(c_j|\mathbf{y}) = \frac{p(\mathbf{y}|c_j)\cdot\pi(c_j)}{p(\mathbf{y})} \quad j = 1,\cdots,mn_t$$

$$APP(c_j) \ \propto \ \pi(c_j)\cdot p(\mathbf{y}|c_j) = \pi(c_j)\cdot obs(c_j)$$

where $\pi(c_j)$ is the *a priori* probability of the bit c_j and the observation $obs(c_j) = p(\mathbf{y}|c_j)$.
Marginalization to get the conditional density:

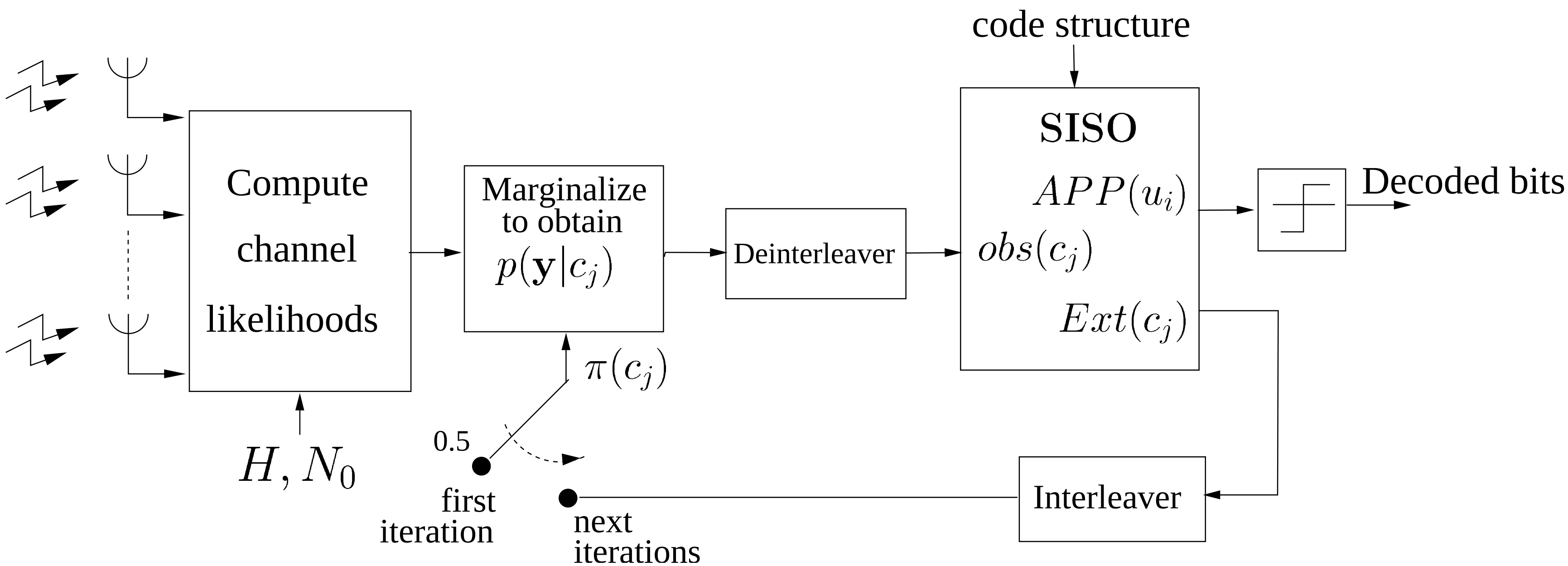
$$p(\mathbf{y}|c_j) = \sum_{\{c_i\},i\neq j} p(\mathbf{y},c_1,..,c_{j-1},c_{j+1},..,c_{mn_t}|c_j) = \sum_{\{c_i\},i\neq j} p(\mathbf{y}|c_1,\cdots,c_{mn_t})\prod_{l\neq j}\pi(c_\ell)$$

$$=> \quad p(\mathbf{y}|c_j) = \sum_{\{c_i\},i\neq j}\left(\prod_{r=1}^{n_r} p(y_r|c_1\cdots c_{mn_t})\prod_{l\neq j}\pi(c_\ell)\right)$$

The channel likelihoods are evaluated by

$$p(y_r|c_1,\cdots,c_{mn_t}) = \frac{e^{-\frac{\left\|y_r-\sum_{t=1}^{n_t}h_{t,r}x_t\right\|^2}{2\sigma^2}}}{(2\pi\sigma^2)}$$

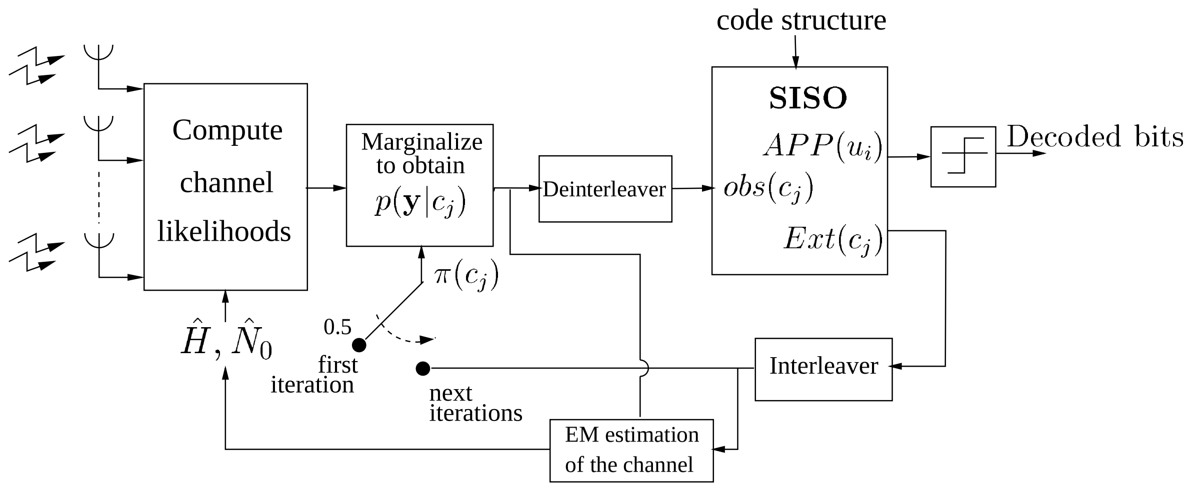
Multiple Antennas Bitwise Receiver



The receiver is separated into two parts:
- the first one is non iterative and computes the received signal conditional probability at every Rx antenn,.
- the second part is iterative and its input depends also on the *a priori* probabilities.

Finally, the decision is made out of the *a posteriori* probability generated by the SISO decoder at the last iteration.

Joint decoding and channel estimation with the EM algorithm



- Parameters to estimate: $\Theta = (N_0, H)$.

- EM algorithm, refining iteratively its estimation Θ^i at step-i working with the function $Q(\Theta|\Theta^i)$ equal to the log-likelihood of the observation $\mathbf{y}$ conditionally to the emitted vectors $\mathbf{x}$ with going through the two following stages
  - stage E ("Expectation"), with the derivation of $Q(\Theta|\Theta^i)$
  - stage M ("Maximization"), with the research of value Θ^{i+1} maximizing $Q(\Theta|\Theta^i)$

Determination of $\Theta = (N_0, H)$

Expectation step: at iteration i, $Q(\Theta|\Theta^i) = E_{\mathbf{x}}\left[\log\left(p(\mathbf{y}|\mathbf{x},\Theta)\right)|\mathbf{y},\Theta^i\right]$

$$=> \quad Q(\Theta|\Theta^i) = -\sum_{k=1}^{\frac{Nc}{mn_t}}\sum_{u=1}^{\mu}\left(2\log(N_0) + A + \frac{||\mathbf{y}(k) - H\mathbf{x}_u||^2}{2N_0}\right) APP_k(\mathbf{x}_u|\Theta^i)$$

Maximization step: find Θ^{i+1} that maximizes $Q(\Theta)$ by deriving $Q(\Theta|\Theta^i)$ with respect to Θ two components:

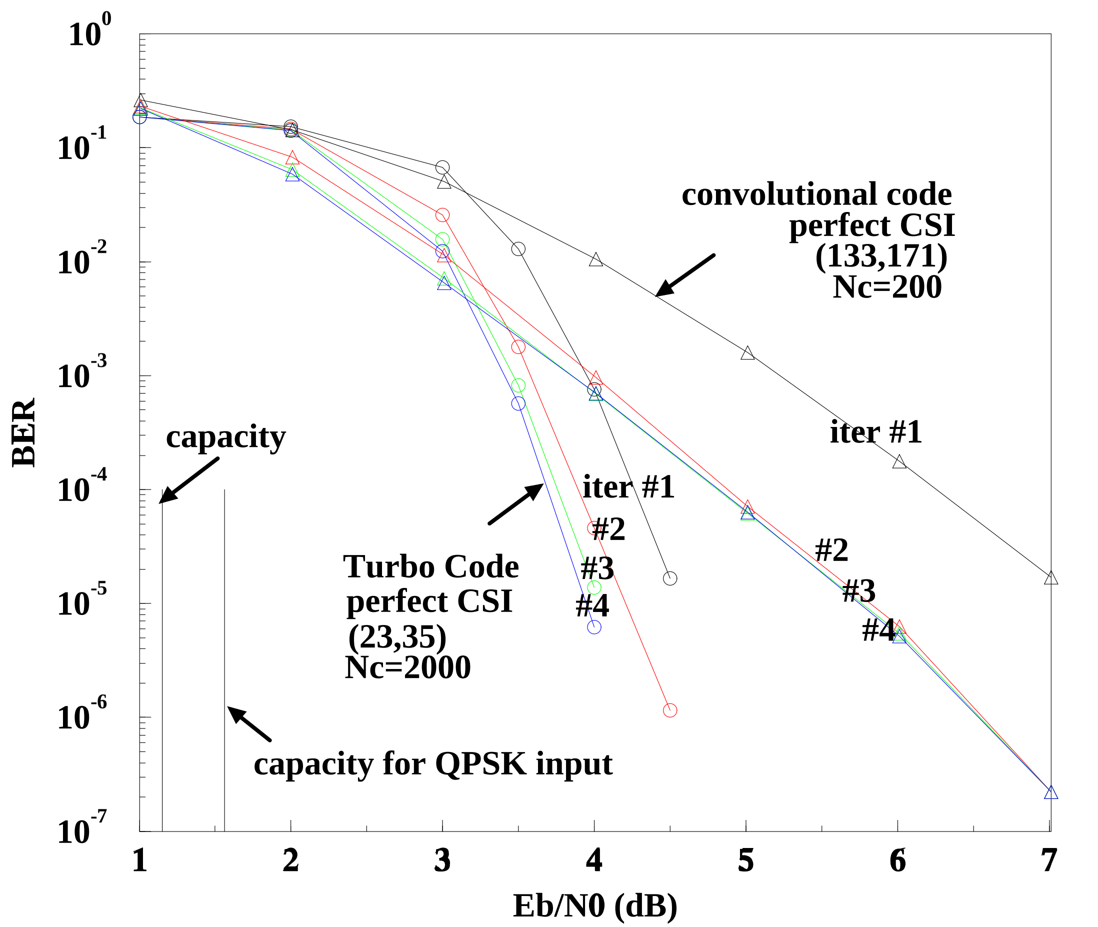
$$H^{i+1} = \sum_{k=1}^{\frac{Nc}{mn_t}}\sum_{u=1}^{|\mathcal{X}|}\mathbf{y}(k)\mathbf{x}_u^h APP_k(\mathbf{x}_u|\Theta^i) \times \left(\sum_{k=1}^{\frac{Nc}{mn_t}}\sum_{u=1}^{|\mathcal{X}|}\mathbf{x}_u\mathbf{x}_u^h APP_k(\mathbf{x}_u|\Theta^i)\right)^{-1}$$

$$N_0^{i+1} = \frac{1}{4\frac{Nc}{mn_t}}\sum_{k=1}^{\frac{Nc}{mn_t}}\sum_{u=1}^{|\mathcal{X}|} APP_k(\mathbf{x}_u|\Theta^i)\cdot||\mathbf{y}(k) - H^{i+1}\mathbf{x}_u||^2$$

☞ The channel parameters estimation is naturally embedded in the APP decoding, as it depends only of channel observations and *a posteriori* probabilities given by the SISO decoder.

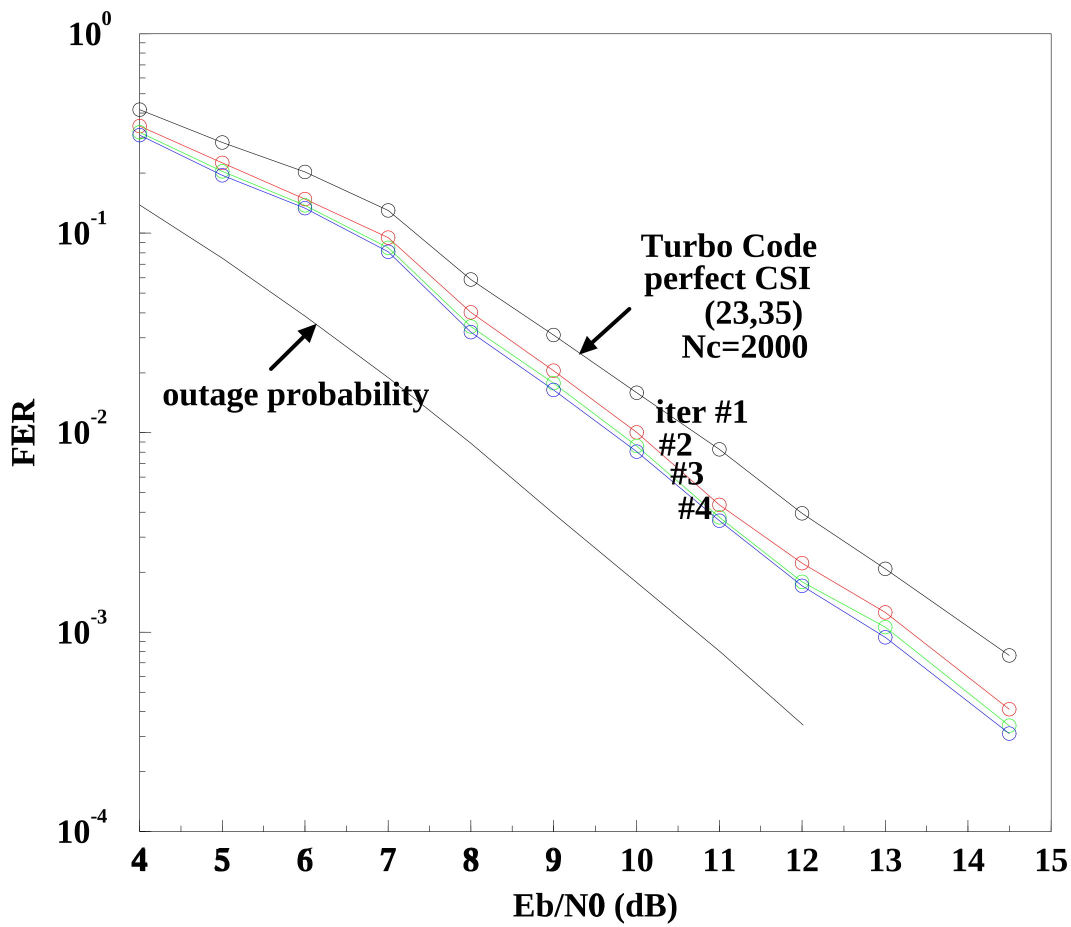
The initial values H^0 dans N_0^0 are obtained with pilot symbols.

Non Static Rayleigh Fading Channel



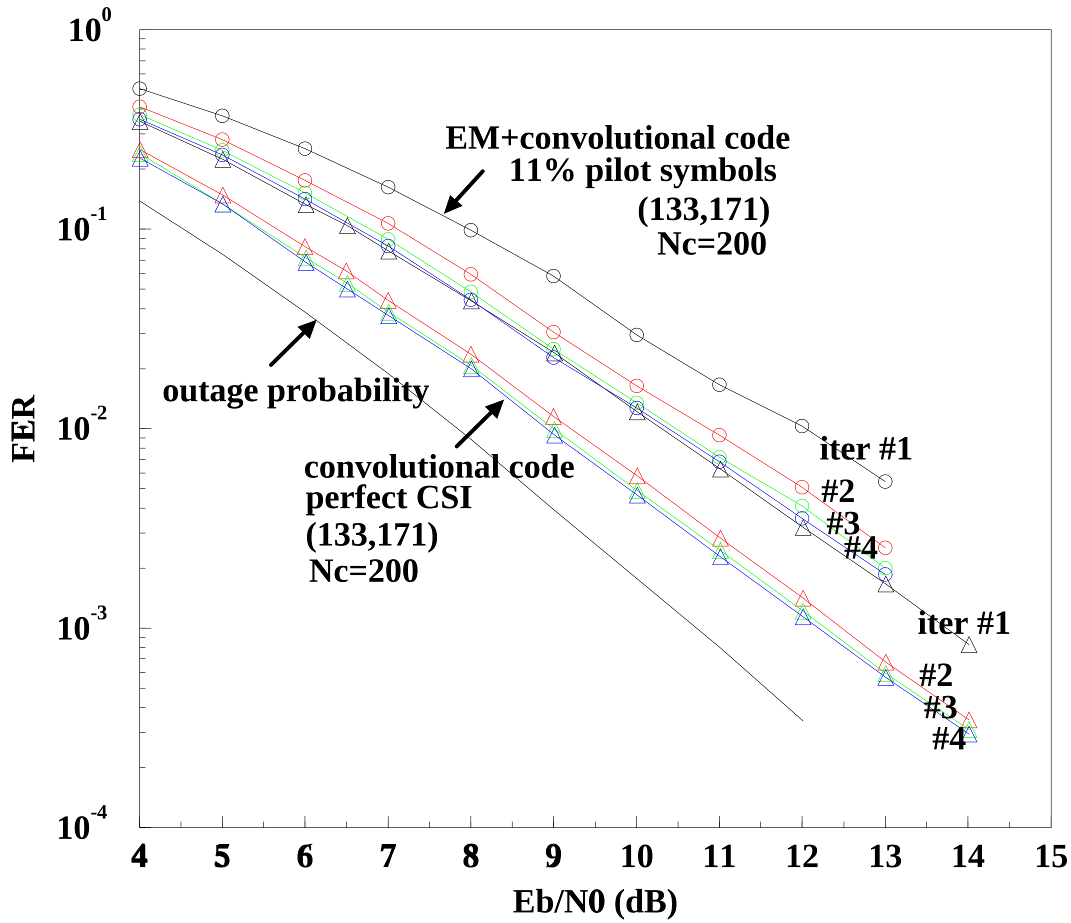
Bit error rate for turbo and convolutional codes, $n_t = n_r = 2$ antennas.

Block Fading Channel



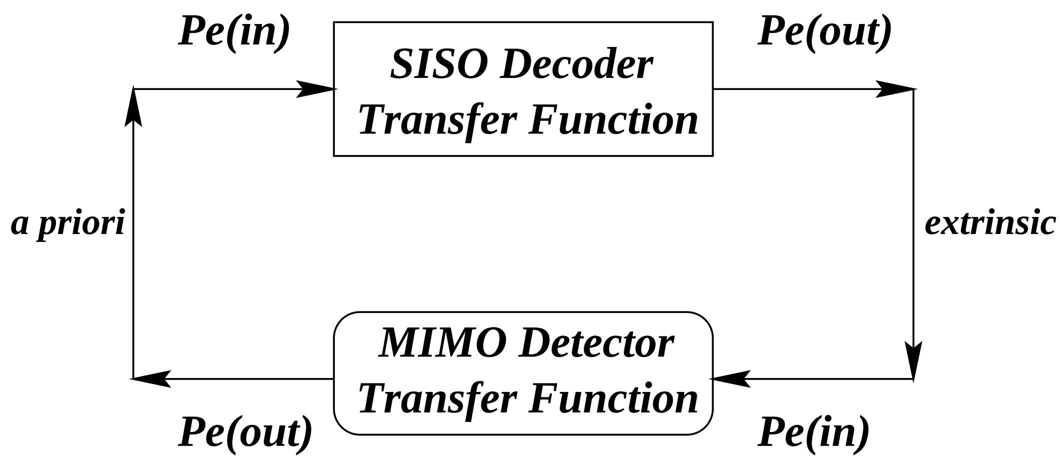
Frame error rate of a turbo code, RSC constituent (23, 35), $n_t = n_r = 2$ antennas.

Block Fading Channel (cont.)



Frame error rate of a convolutional code, generators (133, 171), $n_t = n_r = 2$ antennas.

The Transfer Function Method

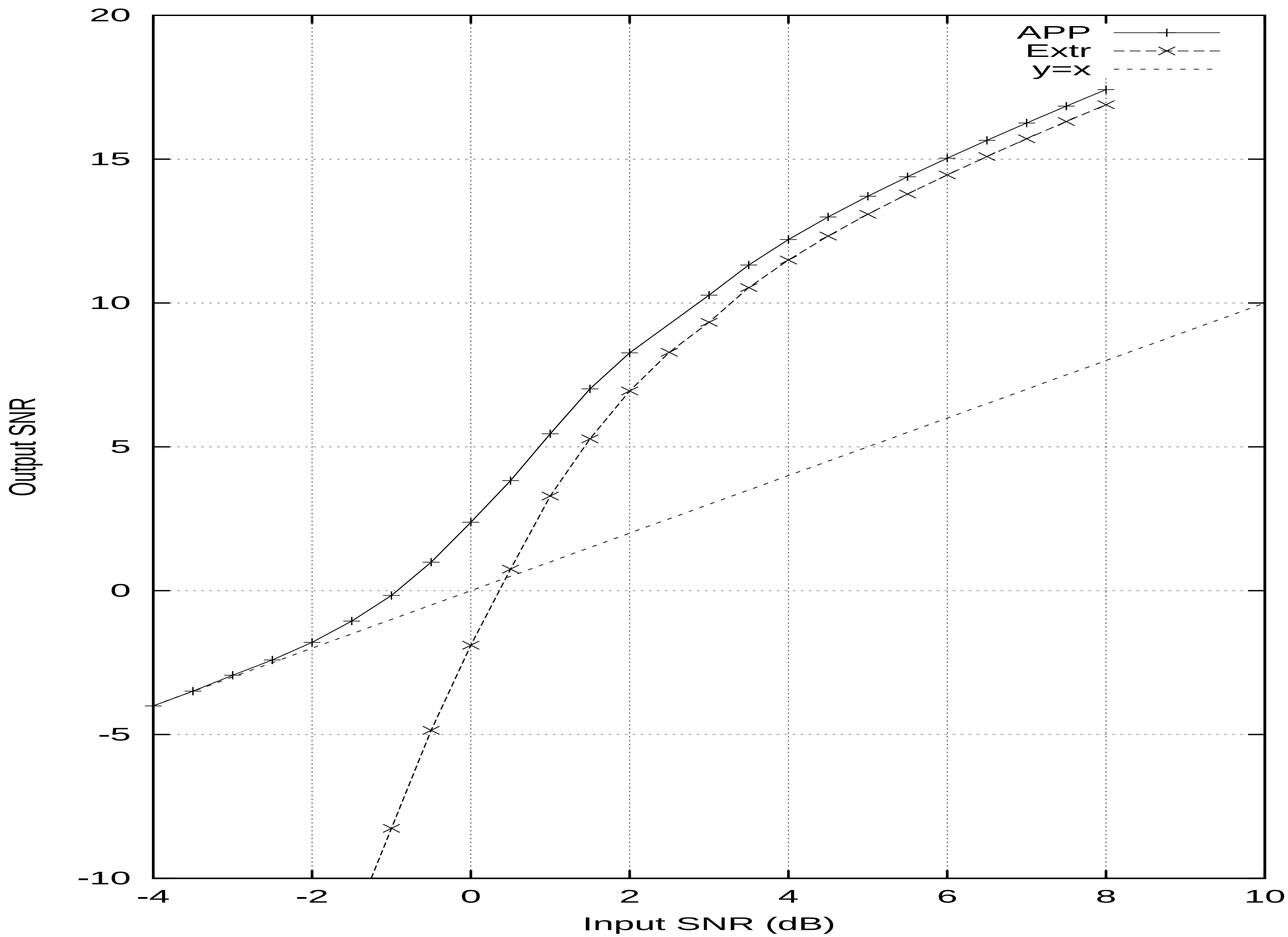


Convolutional code: $Pe_{out} = H(Pe_{in})$ or equivalently $SNR_{out} = H(SNR_{in})$
MIMO signal to APP converter: $Pe_{out} = G(Pe_{in})$ or equivalently $SNR_{out} = G(SNR_{in})$

Fixed point: $SNR = H(G(SNR))$ *i.e.* $H(SNR) = G^{-1}(SNR)$.

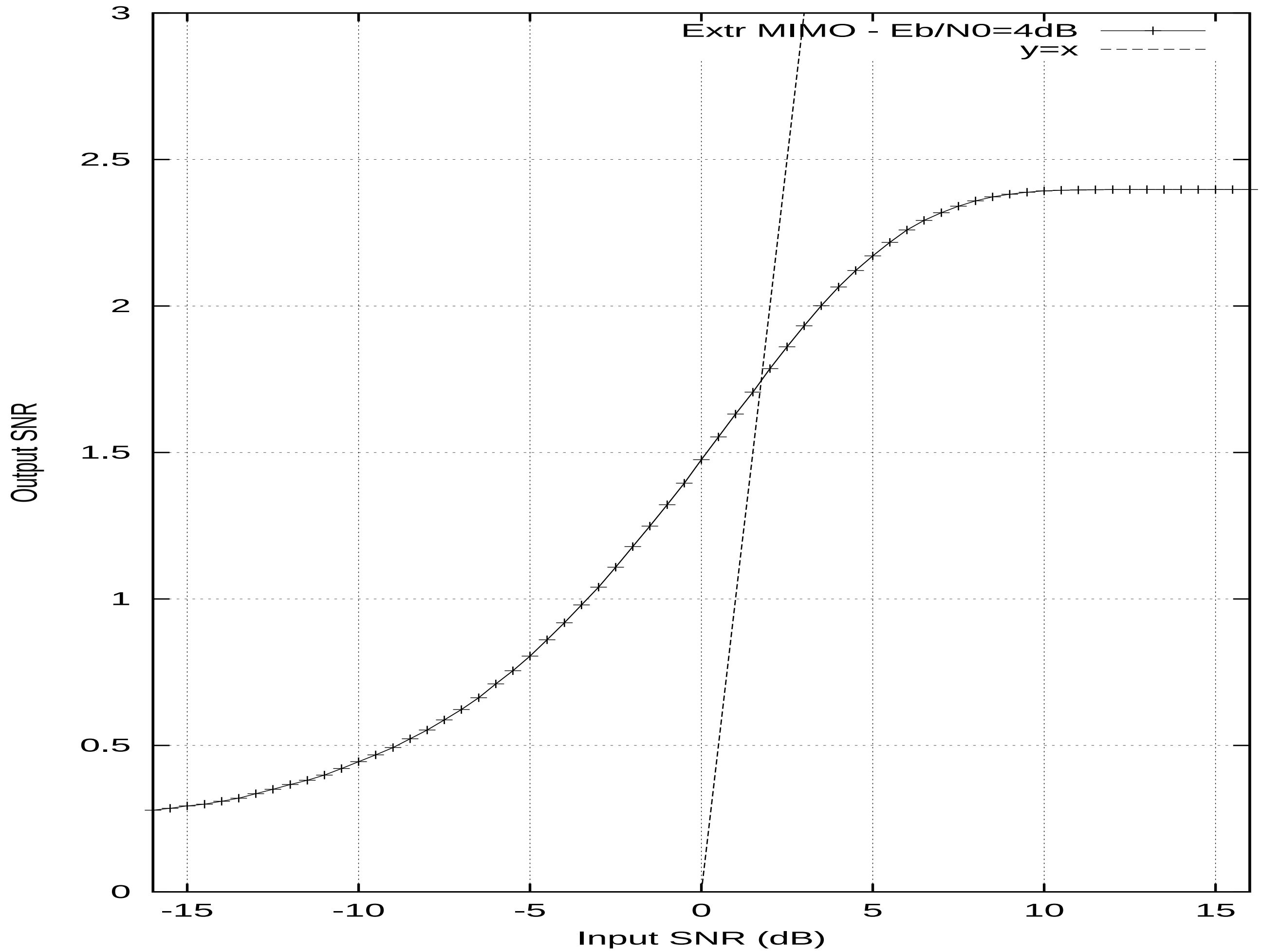
One-to-one mapping between the extrinsic distribution and the bit error probability
$\Longrightarrow$ Gaussian Approximation where the extrinsic log ratio is gaussian distributed with mean $\pm 2/N_0$ and variance $4/N_0$. We get $Pe = Q(\frac{1}{\sqrt{N_0}}) = \frac{1}{2}erfc(\frac{1}{\sqrt{4N_0}})$.

Convolutional Code Transfer Function



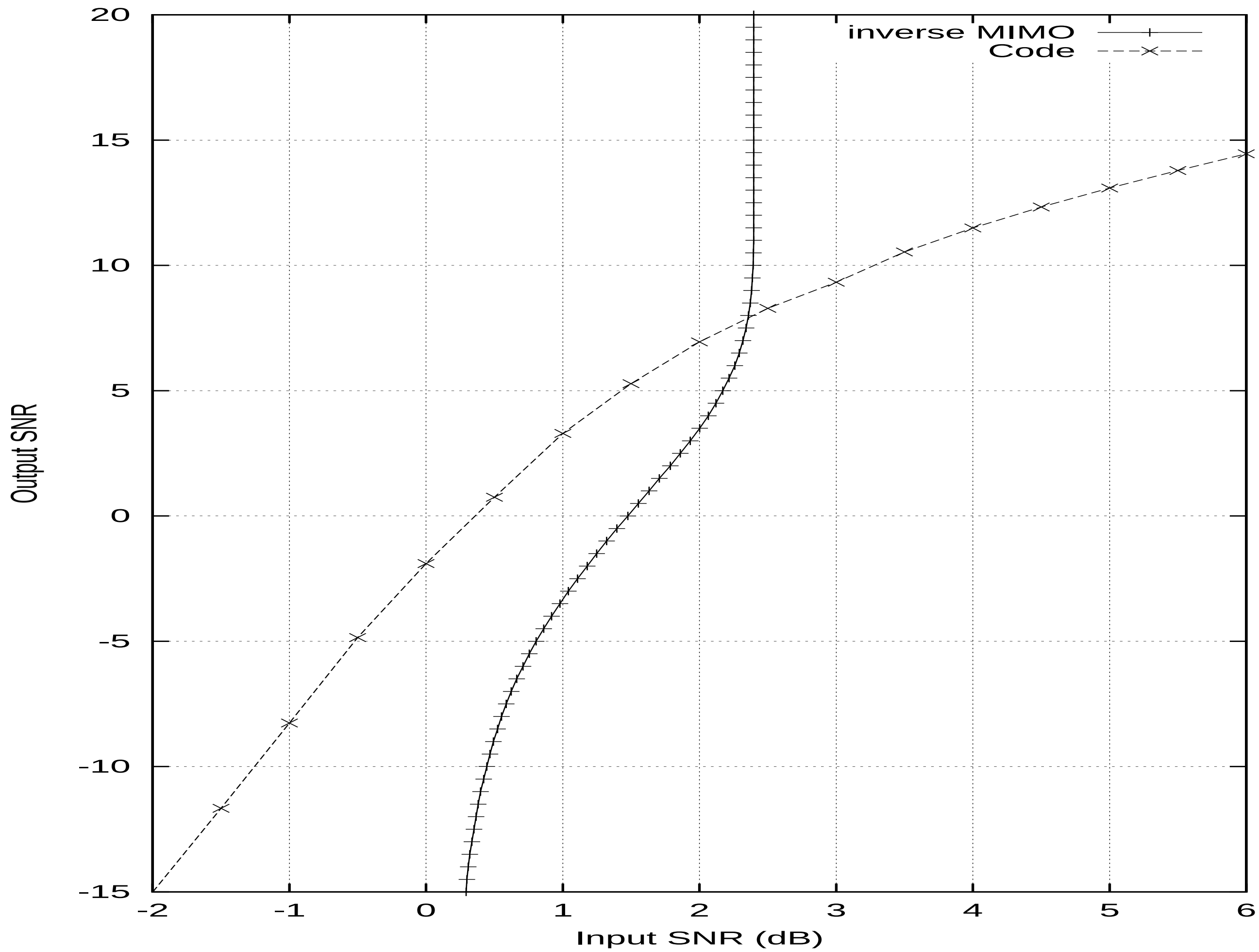
Transfer function of a 64-state rate 1/2 convolutional code.

APP Converter Transfer Function



Transfer function of a MIMO channel detector, $Eb/N0 = 4\ dB$.

Iterative Decoding Fixed Point (asymptotic performance)



Intersection of the code transfer function and the inverse detector transfer function.

Conclusions

- Simple iterative detection of the BICM over the MIMO channel

- The estimation of the channel parameters with the EM algorithm can be integrated into the detection process.

- A small turbo code, interleaver size 1000 and total rate 1/2, exhibits a bit error rate at 2.5 dB distance from the capacity limit.

- A 64-state rate 1/2 convolutional code performs 1.5 dB from outage capacity with perfect channel state information and 3 dB with EM estimation on the block fading channel.

- The transfer function method proves that a small number of iterations is needed ($<= 4$) and that a small finite size interleaver is sufficient.