ISO/IEC JTC 1/SC 29/WG 1 N 3084

Date: 2003-10-27

ISO/IEC JTC1/SC29/WG1
(ITU-T SG8)

Coding of Still Pictures

JBIG	JPEG
Joint Bi-level Image Experts Group	Joint Photographic Experts Group

TITLE: **Selecting default RS codes for EPB marker: RS performances with discussion on GF(16) and GF(256)**

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PROJECT: **JPEG 2000 part 11**

STATUS:

REQUESTED ACTION: **To be presented at WG1 JPWL intermediate meeting in Lausanne, November 2003**

DISTRIBUTION: **WG1 web pages**

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Selecting default RS codes for EPB marker: RS performances with discussion on GF(16) and GF(256) [1]

1. Introduction

During the establishment of JPEG 2000 standard [2], a set of error resilience tools have been selected, for the transmission of JPEG 2000 compressed images in an error prone environment. Two types of tools are available, on the packet level, which enable synchronisation, and on the entropy coding level, enabling error detection. These tools are however based on one major hypothesis, namely that the headers (Main Header and Tile-part(s) header(s)) of the codestream syntax are guaranteed to be error free. However, in the case of error within the headers, the codestream is not decodable in a proper way, which might conduct to a decoder application crash. The worse is that, generally, it might not be possible to guarantee that the headers will be kept free of errors in many applications. In order to extend the error resilience to headers and avoid (or strongly limit) the decoding crashes due to error presence, a new marker called EPB for Error Protection Block was proposed to JPEG 2000 Wireless [3]. It consists of the introduction of a backward compatible error protection mechanism for protecting Main Header and Tile-part(s) header(s) based on Reed-Solomon codes [4].

The purpose of this contribution is to provide elements to select the default Reed Solomon codes needed for the EPB markers, while taking into account the characteristics of JPEG 2000 codestream (e.g. byte alignment), and the bursty comportment of many wireless channels such as our reference the DRM [1] channel.

2. RS codes

2.1. Presentation of Reed-Solomon codes

Reed-Solomon codes are BCH codes over GF(q). Well known in the digital coding world, those linear block codes are widely used because they are the natural codes to use when a code is required of length less than the size of the field for they are maximum distance separable. They are used in digital communications and storage to correct errors in many systems including: storage devices (CD, DVD, barcodes, *etc.*), satellite communications, DVB, high-speed models (ADSL, xDSL, *etc.*) and wireless or mobile communications (cellular phones, microwave links, *etc.*).
Reed-Solomon codes present also the advantage of being easily implementable, both for coding and decoding, and a number of commercial software and hardware implementations of these codes is available, including *off-the-shelf* integrated circuits and VHDL components that encode and decode Reed-Solomon codes.
This explains why it was proposed in [3] to consider RS codes as default FEC codes for header protection of JPEG 2000 over wireless links.

The Reed-Solomon encoder takes a block of digital data (typically k bytes) and generate n-k redundancy bytes that may be placed after the k original (systematic) bytes, this process being applied as long as necessary, as illustrated in Figure 1. The RS decoder processes then each received block and attempts to correct errors that have occurred during transmission or storage due to noise or interference to recover the original data. The number and type of errors that can be corrected depends on the characteristics of the Reed-Solomon code.

A Reed-Solomon code RS(n,k) is defined over a Galois Field GF(q). Generally, q is taken equal to 2^m corresponding to RS symbols of m bits. This means that the RS encoder takes groups of k*m bits and adds (n-k) parity symbols corresponding to (n-k)*m parity bits. The RS decoder can then correct up to t erroneous symbols, where 2t=n-k.

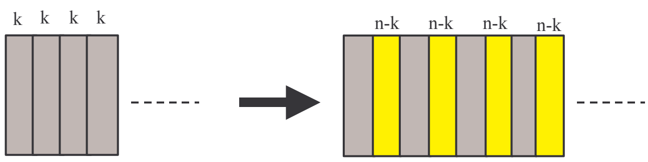

Figure 1 - Example of redundancy generation for an RS(n,k) code.

[1] This work was supported by the European Commission through the European project 2KAN (IST-2001-34096).
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2.2. Derivation of RS codes theoretical performances over the BSC

By definition an RS(n,k) code over GF(2^m) allows to correct t=(n-k)/2 erroneous symbols. Considering a Binary Symmetric Channel (BSC) with binary error probability p, let us determine the bit error probability of such a code.

First, let consider the error probability P_{es} of the Reed-Solomon symbol over the channel. It is easy to see that it can be expressed as: $P_{es} = 1-(1-p)^m$ where p is the binary error probability over the BSC.
A decoding error of the RS code will then correspond to the case were there will be more than t errors in the n transmitted symbols, so the error probability of our code is given as the sum of all cases where there is more than t+1 errors among n, namely by $P_e = \sum_{i=t+1}^{n} \binom{i}{n} P_{es}^i (1-P_{es})^{n-i}$, which can be also expressed as:

$$P_e = P_{es}^n \sum_{i=0}^{n-t-1} \binom{i}{n} \left(\frac{1-P_{es}}{P_{es}}\right)^i, \; with \; P_{es} = 1-(1-p)^m \; and \; t = \frac{n-k}{2}. \qquad (1)$$

This formula allows us to draw the theoretical performance of various Reed-Solomon codes, and consequently to determine which codes we should choose based on the foreseen bit error probability on the BSC channel. This theoretical symbol error rate corresponds to the probability of not having a good decoding of the RS block of n bytes. The corresponding curves are consequently expressed in Packet Error Rate (PER), and are to be regarded as upper bounds for Symbol and Bit Error Rate (SER and BER). Figure 2 gives the performances of various RS(128,x) codes over GF(256), Figure 3 provides similar results for RS(255,x) codes over GF(256) and Figure 4 gives the theoretical performances of all possible RS codes over GF(16) (considering that RS(15,2k) and RS(15,2k+1) have same performance).

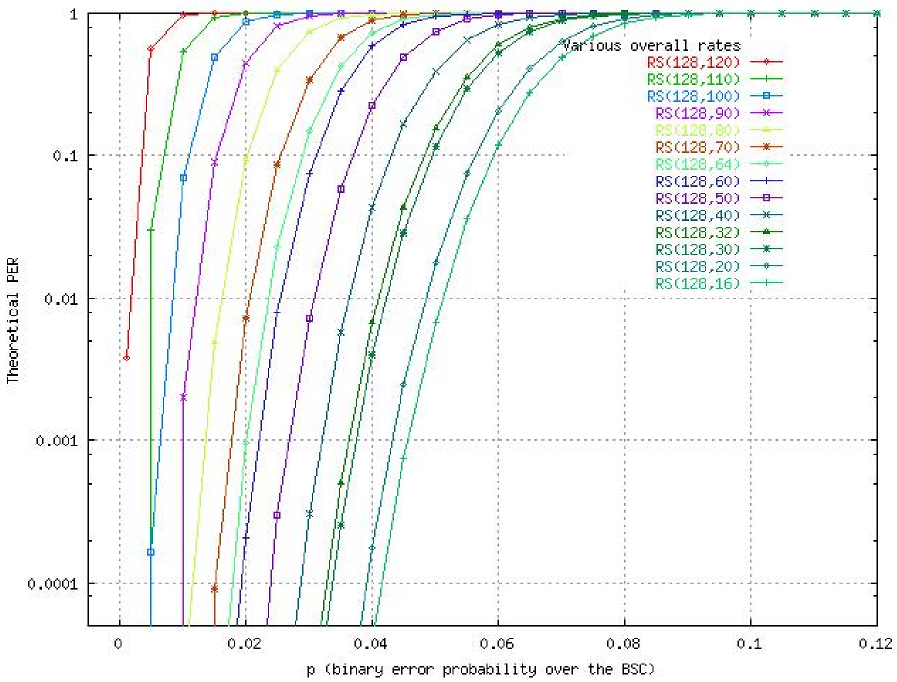

Figure 2 - Performances of RS(128,x) codes over GF(256).

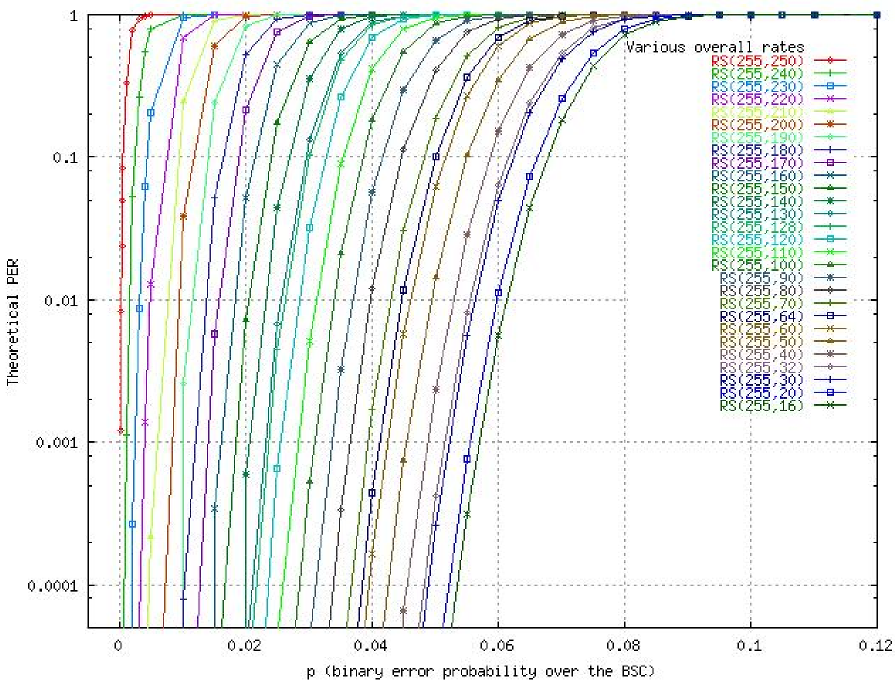

Figure 3 - Performances of RS(255,x) codes over GF(256).

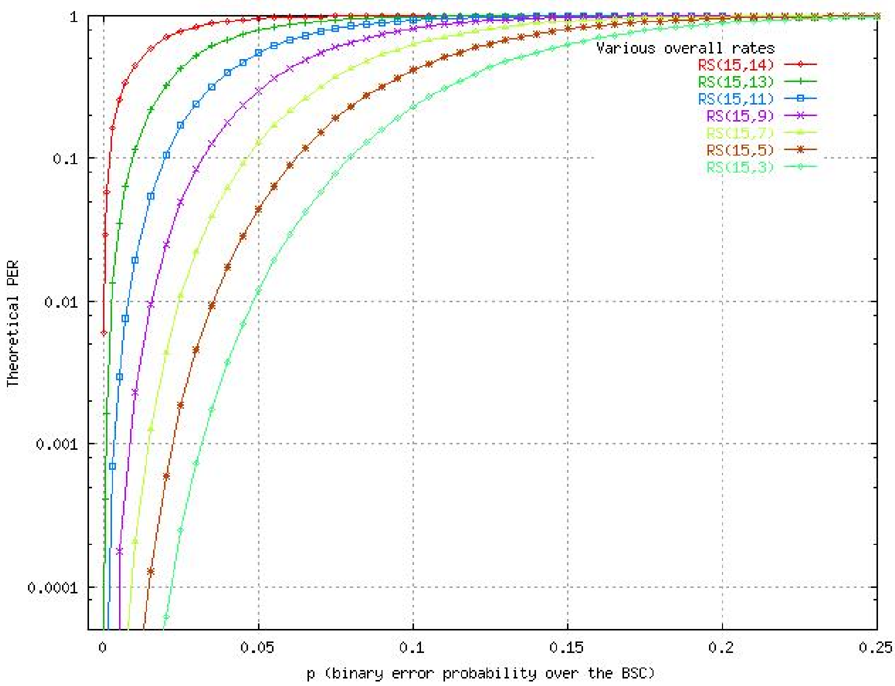

Figure 4 - Performances of RS(15,x) codes over GF(16).

2.3. Comparison performances between GF(16) and GF(256) over the BSC

Considering that JPEG and JPEG 2000 codestreams are byte aligned, it is especially interesting to work with Reed-Solomon codes operating over Galois Field GF(2^m) where m is either a divisor or a multiple of 8. As a matter of fact, such a choice will prevent any byte alignment complication and propagation of errors to bytes not protected by the same RS block of symbols. In practice, this leads to either m=4 or m=8.

Considering Equation (1), it is easy to understand that for the same (n,k) values, P_{es} increases with m, and that it seems consequently better to work on the smallest Galois Field possible for a given set of code length and dimension. This is illustrated in Figure 7 by the respective performances of RS(15,7) and RS(15,5) over GF(256)

and over GF(16): it can be observed that codes over GF(256) perform systematically worse than those over GF(16).
As foreseen, it can also be noted that the smaller the code rate k/n, the better the code performance. This corresponds to an increase of the size of the RS redundancy and is easily justified by Equation (1) for n constant considering that a smaller code rate leads to a better the correction capacity t. As an example, the respective performances of rate 1/2 and rate 2/3 codes.
This being said, a comparison remains to be done for codes with different k, n and m parameters. Indeed, the same coding rate k/n can be achieved with various (k,n) pairs, and Figure 7 illustrates this for three different cases: for n in {15, 128, 255} and k/n in {1/2, 2/3}. It can be observed that the larger codes perform better when p is small and that the smaller codes perform better when p is large.

Now, let us explain this by interpreting the error probability P_e.
Let consider a noisy RS symbol. It consists of m noisy bits, each being a random variable X which are independent and identically distributed (i.i.d.) . Note that the bit values may be correlated but that the impact of noise made them independent. Random variable X first moments are: E(X)=p and $E(X^2)=p^2$.
The noisy symbol S can then be seen as a random variable S i.i.d. with two possible values: S=1 corresponding to an error with $p(S=1)=1-(1-p)^m$ and S=0 corresponding to no error with $p(S=0)=(1-p)^m$. Its first moments are then given by:

$$E(S)=1-(1-p)^m \ ,$$
$$E(S^2)=1-(1-p)^m \ ,$$
$$\sigma_X^2=(1-p)^m-(1-p)^{2m} .$$

The central limit theorem tells us then that the sum $S_n=\sum_{i=1}^{n} S^i$ of I independant realisations of S (subscript ^{i} representing the numbering of said realisation) approaches a Gaussian distribution with the following parameters:

$$\frac{S_n}{n} \underset{n\to\infty}{\rightarrow} \mathrm{N}\left(\left(\overline{\frac{S_n}{n}}\right),\sigma\right)=\mathrm{N}\left(E(S),\frac{\sigma_S}{\sqrt{n}}\right)=\mathrm{N}\left(1-(1-p)^m,\sqrt{\frac{(1-p)^m-(1-p)^{2m}}{n}}\right) \qquad (2)$$

Equation (2) allows us to consider P_e as the tail of a Gaussian distribution whose mean and variance depend of p and m. The dependence in k and n appears in the boundary at t/n.

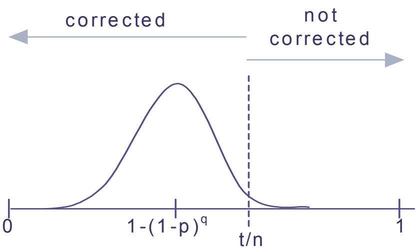

Figure 5 - Illustration of Equation (2): representation of Pe as a Gaussian tail.

Conceptually, if we suppose that Equation (2) holds for n values equal to 15 and 255, the corresponding Gaussian distributions $N(\mu_{15},\sigma_{15})$ and $N(\mu_{255},\sigma_{255})$ verify that:

$\mu_{15} \le \mu_{255}$,

for small values of p (e.g. p < 0.01), $\sigma_{15} \ge \sigma_{255}$,

for large values of p (e.g. p > 0.2), $\sigma_{15} \le \sigma_{255}$.

An illustration of the corresponding distribution is given in Figure 6 for a small value of p.

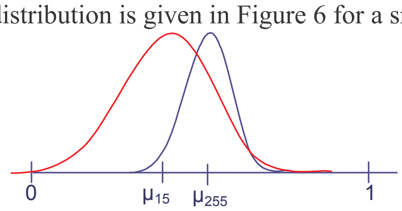

Figure 6 - Comparison between distributions $N(\mu_{15},\sigma_{15})$ and $N(\mu_{255},\sigma_{255})$.

Those results are illustrated by those obtained in Figure 7, and led us to the following conclusion: for error correction levels leading to packet error rate below 10^{-2}, *i.e.* for if binary error rate over the BSC channel p small enough (typically $p<2.10^{-2}$), one should consider RS codes over GF(256). Should larger error rates after RS decoding be accepted, RS codes over GF(16) can then be recommended.

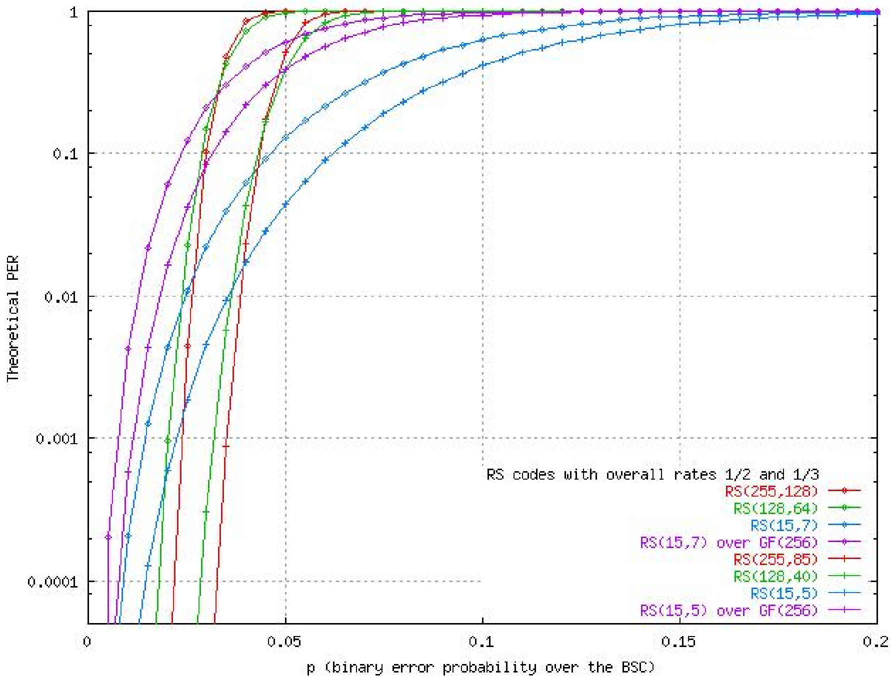

Figure 7 - Performance comparison for several rate 1/2 and 2/3 over GF(16) and GF(256).

2.4. Simulation results over DRM channels and discussion

Considering now the case of DRM channels, the Gilbert modelisation retained for these channels as two BSC channels linked by a two-state Markov chain leads us to considerations similar to those exposed for the simple BSC case. As a matter of fact, the GOOD channel being a perfect transmission channel, the only impacts in terms or decoding error come from the BAD channel, which is a BSC. The binary error probability e_b on this BAD channel corresponds to binary error rate p considered in the previous section.

3. Conclusions

RS codes over GF(256) are recommended as default codes for EPB marker segment.

4. References

[1] Digital Radio Mondiale (DRM): System Specification. ETSI ES 201 980. Jul. 2002.

[2] *JPEG 2000 image coding system.* ISO/IEC 15444-1/ IUT-T T.800.

[3] D. Nicholson, C. Lamy, C. Poulliat and X. Naturel, "Result of Core Experiments on 'Header error protection' (JPWL C01)", *ISO/IEC JTC1 SC29 WG1* N2935, Lausanne, Switzerland, May 2003.

[4] F.G. MacWilliams, N.J.A Sloane, *The Theory of error correcting codes: Part 1*, North-Holland Publishing Company: New York, 1977, pp. 294-315.

[5] D. Nicholson, C. Lamy-Bergot, and X. Naturel, "Transmission of JPEG 2000 images over a DRM system: error patterns and modelisation of DRM channels", *ISO/IEC JTC1 SC29 WG1* N3083, Lausanne, Switzerland, Nov. 2003.

5. Acknowledgements

The authors wish to thank M. Nicolas Marguinaud for his insights and precious comments.