On Random Rotations Diversity and Minimum MSE Decoding of Lattices

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Abstract

We establish a simple relation between high diversity multidimensional rotations obtained from totally complex cyclotomic fields and the discrete Fourier transform. The diversity distribution of an Hadamard-like random rotation is derived analytically. It is shown that a random multidimensional rotation exhibits an excellent diversity distribution and can be combined to QAM constellations to combat channel fading. We also describe a mean square error (MSE) universal lattice decoder suitable for large dimensions up to 1024. The MSE criterion treats the lattice structure as intersymbol interference. The universal decoder is applied to both Gaussian and Rayleigh fading channels to decode dense lattice sphere packings and rotated cubic constellations respectively.

Key Words : Random Lattices, Diversity, Decision Feedback Detection

1 Introduction

A lattice is a discrete subgroup of rank n in the real n-dimensional space $\mathbb{R}^n$ [1]. Dense lattice constellations are a good mean to achieve a reasonable coding gain on the Gaussian channel with high spectral efficiency. The maximum likelihood (ML) decoding of a lattice Λ is equivalent to finding the lattice point $\mathbf{x}$ at a minimum Euclidean distance to the received point $\mathbf{r}$. This task is too complex especially for large dimensions. Integer lattices built by constructions A and B can be decoded with a multistage technique based on soft decision decoding of the constituent linear binary codes [1], [2]. Many decoding algorithms for the Gaussian channel are also known for some dense integer lattices [2], [3], [4]. Recently, a sub-optimal algorithm (generalized minimum distance algorithm, GMD) for integer lattices [5] has been described for decoding over the Gaussian channel. All these algorithms have been developed for the Gaussian channel and they are mainly based on the soft decision decoding of linear codes.

Firstly presented by Boullé & Belfiore [6] in the two-dimensional real space, and then extented to dimensions up to 5 in [7], the idea of rotating a quadrature amplitude modulation (QAM) constellation was proven to increase the diversity order on the flat fading Rayleigh channel by spreading the information contained in each component over several components of the constellation points. Multidimensional algebraic rotations have been recently derived [8] in a real or a complex space with half or full diversity orders and no limit on the space dimension. Algebraic number theory has been also applied to construct coded and uncoded lattice constellations with high diversity orders [9], [10]. Kerpez proposed integer high diversity constellations [11] which can be used to combat fading, but as shown in [7] they do not outperfom rotated cubic constellations due to their large average energy per component.

The coherent ML decoding on a Rayleigh channel is equivalent to the minimization of $||\mathbf{r}-\alpha * \mathbf{x}||^2=\sum_{i=0}^{n-1}|r_i-\alpha_i x_i|^2$, where $\{\alpha_i\}$ are the real Rayleigh distributed fading coefficients. For this channel, the lattice density producing a positive gain on the Gaussian channel has no more effect. The performance of a lattice on the Rayleigh fading channel depends on its diversity given by the Hamming distance distribution of the lattice points. High diversity rotated integer and integral lattices cannot be decoded using Gaussian channel algorithms mentioned above. Universal sphere decoding of lattices [12] is a mean to decode any lattice on both Gaussian and Rayleigh fading channels. This algorithm is maximum likelihood but its complexity limits its use to dimensions less than or equal to 32.

In this paper, we study the diversity distribution of Hadamard, Fourier and random orthogonal transforms (i.e. multidimensional rotations) and describe a lattice decoder based on the minimization of the mean square error for the decoding of high dimensional constellations. Firstly, a simple relation is established between high diversity multidimensional rotations obtained from totally complex cyclotomic fields [8] and the discrete Fourier transform. Further, the diversity distribution of random ± 1 Hadamard-like matrices is derived analytically. Then, we describe a sub-optimal universal algorithm for decoding an arbitrary lattice Λ in dimensions up to 1024 for both Gaussian and Rayleigh fading channels. This algorithm is based on the Minimum Mean-Square-Error (MSE) criterion. Instead of minimizing the Euclidean distance, the MSE decoder minimizes the expectation of the squared error in the integer space $\mathbb{Z}^n$ before applying the lattice generator matrix. The first version of an MSE equalizer (as a decision feedback equalizer DFE) has been presented in [13] for decoding Hadamard and Fourier matrices over the Rayleigh fading channel. This paper generalizes the MSE decoding for any n-dimensional real or complex lattice. The excellent performance obtained in [13] are explained by the lattice diversity distribution (see section 3). Surprisingly, in the fading channel case, rotated lattices selected at random perform as good as those built algebraically, e.g. rotated versions of the cubic lattice $\mathbb{Z}^n$ denoted by $\mathbb{Z}_{n,L}$ where L is the lattice diversity order.

The paper is organized as follows : Section 2 gives the definition of the lattice gain on the Gaussian channel and the lattice diversity on the Rayleigh fading channel. Section 3 describes the relation between algebraic rotations and fast transforms. The diversity distributions of random ± 1 matrices, Fourier matrices and $\mathbb{Z}_{n,n/2}$ rotations are compared in this section. Section 4 presents the MSE decoding of lattices based on decision feedback block equalization. Section 5 shows the performance of the MSE decoder applied to the Barnes-Wall lattice BW_{256} and the rotated lattice $\mathbb{Z}_{512,256}$ (equivalent to a complex 256-dimensional rotation) and finally section 6 draws out the conclusions.

2 The performance of a lattice code : density and diversity

The system model using a lattice constellation is shown in Fig. 1. On the Gaussian channel, the point error rate P_e of this system decreases exponentially as the signal-to-noise increases. For a cubic constellation, the error probability is given by [10]

$$P_e \approx \frac{\tau}{2}\, erfc\left(\sqrt{\frac{3s}{2^{s+1}} \times \frac{E_b}{N_0} \times \gamma(\Lambda)}\right), \tag{1}$$

where s is the number of bits per two dimensions and τ is the lattice kissing number. The parameter $\gamma(\Lambda)$ is the fundamental gain of Λ. On the Rayleigh fading channel, the point error rate decreases linearly with a slope of order L [10]

$$P_e \leq \sum_{l=L}^{n} \frac{K_l}{\left(\frac{s}{8}\frac{E_b}{N_0}\right)^l}. \tag{2}$$

The positive constants K_l depend on the choice of the lattice Λ. The lattice diversity L and the fundamental $\gamma(\Lambda)$ are defined below.

The fundamental gain $\gamma(\Lambda)$ of a lattice on the Gaussian channel is due to its high packing density. The gain is given by the following ratio (Hermite's constant, [1] p. 20, pp. 71–74)

$$\gamma(\Lambda) = \frac{d^2_{Emin}}{\sqrt[\frac{n}{2}]{vol(\Lambda)}},$$

where d_{Emin} is the minimal Euclidean distance of Λ and $vol(\Lambda)$ is the fundamental volume. Thus, the gain increases with the lattice density since a higher density means a larger Euclidean distance and a smaller fundamental volume.

The diversity order L of a lattice Λ is the minimum number of distinct components between any two points belonging to Λ, $L = \min_{\mathbf{x},\mathbf{y}\in\Lambda} d_H(\mathbf{x},\mathbf{y})$. Maximizing the diversity is the best way to reduce the error probability on the Rayleigh fading channel.

It has been shown in [8] that a multidimensional rotation increases the diversity of a lattice code. As illustrated in Fig. 2 on a 4-PSK, a simple rotation increases the diversity from $L = 1$ to $L = 2$. A system based on multidimensional rotations that increase the diversity order can be used on the Gaussian channel without any loss in performance.

3 Fast Rotation Transform for fading channels

3.1 From algebraic rotations to Fast Rotations

The non-trivial problem when searching for a multidimensional rotation is to guarantee a diversity order greater than or equal to L. One solution has been proposed in [8] where the n-dimensional real rotation R with diversity order $L = n/2$ is the generator matrix of the rotated cubic lattice $\mathbb{Z}_{n,L} = R\mathbb{Z}^n$. The rotated lattice is built by applying a canonical embedding to the ring of integers in a totally complex cyclotomic number field. The number field is generated by $\theta = e^{2j\pi/N}$ ($n = \phi(N)$ where ϕ is the Euler function). By denoting $\theta_i = \theta \times e^{4j\pi(i-1)/n}$, for $i = 1...n/2$, the rotation matrix is given by the following $n/2 \times n/2$ complex form

$$R = \begin{pmatrix} 1 & 1 & \dots & 1 \\ \theta_1 & \theta_2 & \dots & \theta_{n/2} \\ \vdots & \vdots & & \vdots \\ \theta_1^{n/2-1} & \theta_2^{n/2-1} & \dots & \theta_{n/2}^{n/2-1} \end{pmatrix} \tag{3}$$

This $n/2$-dimensional complex rotation of diversity $n/2$ (full diversity) is equivalent to a real n-dimensional rotation of diversity $n/2$. The $n \times n$ real form can be obtained by replacing each complex entry $a + jb$ of R with a 2×2 matrix $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$.

If we restrict the values of n to powers of 2, we obtain $N = 2n$ and we then evaluate the coefficients r_{ik} of the rotation matrix R by

$$r_{ik} = (\theta.e^{\frac{4j\pi i}{n}})^k = e^{\frac{j\pi k}{n}}.e^{\frac{2j\pi ik}{n/2}} \qquad \forall(i,k) \in 0 \dots \frac{n}{2} - 1. \tag{4}$$

Now, if this rotation is applied to a vector $X = (x_0, \dots, x_{n/2-1})$, the resulting vector is $P = (p_0, \dots, p_{n/2-1})$ given by

$$p_i = \sum_{k=0}^{\frac{n}{2}-1}(r_{ik}.x_k) = \sum_{k=0}^{\frac{n}{2}-1}(e^{\frac{2j\pi ik}{n/2}}.x'_k) \qquad \text{with} \quad x'_k = x_k.e^{\frac{j\pi k}{2(n/2)}}. \tag{5}$$

If we compare the above formula with the Discrete Fourier Transform (DFT), we see that an algebraic rotation based on the lattice $\mathbb{Z}_{n,n/2}$ is equivalent to $n/2$ phase shifts followed by a DFT. Thus, a complex Fast Rotation Transform (FRT) of full diversity is obtained by combining bidimensional rotations and a Fast Fourier Transform (FFT).

The Euclidean distance distribution of a lattice Λ has a direct impact on its performance on the Gaussian channel (asymptotically it depends only on τ and d_{Emin}). In a similar way, the diversity distribution of Λ has a direct impact on its performance over the Rayleigh fading channel. Since $\mathbf{0}$ belongs to Λ, the diversity distribution can be obtained by comparing all points to $\mathbf{0}$. For $\mathbf{x} \in \Lambda$ chosen at random, the diversity distribution gives the probability $P(l)$ that l components of $\mathbf{x}$ are non zero, where $L \leq l \leq n$. The diversity distribution of an FRT (complex form) is simply given by $P(l) = 0$ for $l = 0 \dots n/2-1$ and $P(n/2) = 1$. The minimal diversity order of an FFT is $L = 1$, i.e. $P(1) \neq 0$. This is trivial because $(1, 0, 0, \dots, 0)$ belongs to the FFT lattice. Rigorously, the FFT has no diversity but its diversity distribution approaches the FRT distribution for very large dimensions n (see Fig. 2 in the next section). Thus, for n large enough, the FFT and the FRT have the same performance on the Rayleigh fading channel. The FFT performs badly for $n \leq 32$ whereas an FRT decoded with the universal sphere decoder [12] eliminates almost completely the fading effect. The behavior of FHT (Fast Hadamard Transform) is similar to that of an FFT. In the next sub-section, the diversity distribution of a random Hadamard-like matrix is computed and it is shown that for n large enough FRT, FFT, FHT, and even any rotation chosen at random perform similarly on the Rayleigh fading channel.

3.2 Diversity distribution of random Hadamard-like matrices

Let us consider a spectral efficiency of 1 bit per dimension ($s = 2$). In this case, the Hadamard matrix (i.e. the generator matrix of Λ) is multiplied by an integer vector u whose components take the values 0 or 1. The Hadamard matrices are classically computed recursively [14] pp. 44–49

$$H_1 = 1, \quad H_{2n} = \frac{1}{\sqrt{2n}} \begin{pmatrix} H_n & H_n \\ H_n & -H_n \end{pmatrix}$$

An exact formula for the entries h_{ij} of a Hadamard matrix is very difficult to express. Hence, we define a random Hadamard-like matrix H_n that will enable us to compute the diversity distribution:

- the first row and the first column of H_n are filled with *'1'*.
- the other rows of H_n are composed by $n/2$ *'1'* (including the one in the first column) and $n/2$ *'-1'* equally distributed.
- we voluntarily forget the scaling factor $\frac{1}{\sqrt{n}}$.

Let us define:

- u the input vector, and v the output vector ($v \in \Lambda$), such that $v = Hu$
- $U_{k|0}$ the set of vectors u with k components equal to *'1'* and $u_0 = 0$. Its size is $|U_{k|0}| = \binom{n-1}{k}$
- $U_{k|1}$ the set of vectors u with k components equal to *'1'* and $u_0 = 1$. Its size is $|U_{k|1}| = \binom{n-1}{k-1}$
- For U being one of these two sets and for $d \in \mathbb{Z}$, we define

$$P_U(d) = P(v_i = d \mid u \in U) = P(v_j = d \mid u \in U) \quad \forall (i,j) \in \{1, ..., n-1\}^2. \tag{6}$$

  Note that $v_0 \neq 0$ for any $u \neq 0$.

- $L[l]$ is the number of vectors with diversity l.

With these notations, the number of vectors v with diversity l while $u \in U$ is

$$L[l]_U = |U| P_U(0)^{n-l}(1 - P_U(0))^{l-1}. \tag{7}$$

This gives us the general formula of the diversity distribution of these random matrices

$$\forall l \in \{1, ..., n-1\}$$
$$L[l] = \sum_{k_1'=1}^{n/2} \left(\binom{n-1}{2k_1'} P_{U_{2k_1'|0}}(0)^{n-l}(1 - P_{U_{2k_1'|0}}(0))^{l-1} + \binom{n-1}{2k_1'-1} P_{U_{2k_1'|1}}(0)^{n-l}(1 - P_{U_{2k_1'|1}}(0))^{l-1} \right)$$

and

$$L[n] = \sum_{k_1'=1}^{n/2} \left(\binom{n-1}{2k_1'} (1 - P_{U_{2k_1'|0}}(0))^{n-1} + \binom{n-1}{2k_1'-1} (1 - P_{U_{2k_1'|1}}(0))^{n-1} + \binom{n}{2k_1'-1} \right) \tag{8}$$

Similar expressions can be derived for a higher spectral efficiency. Fig. 3 shows the diversity distribution, for complex 512-dimensional and 8-dimensional FRT, complex 8-dimensional and 512-dimensional FFT and real 8-dimensional and 512-dimensional FHT. As illustrated, Hadamard matrices (or a random ± 1 matrix) have the worst diversity distribution. Thus, the rotation matrix must be properly chosen for low dimensions, e.g. $N = 8$. However, when N is large enough, the three distributions sketched in Fig. 3 (b) exhibit high diversity orders. Note that the diversity scale starts at 500. This explains why all these rotations show practically the same error rate for dimensions larger than 256 on the Rayleigh fading channel.

The diversity distribution given by equations (8) is analytically tractable because the matrix entries are limited to two values on the unity circle, i.e. ± 1. A straightforward generalization is to choose the matrix entries from a points uniformly distributed on the unity circle, $a \geq 2$. A Gram-Schmidt procedure is applied to the random matrix to guarantee its orthogonality. When $a > 2$, finding an explicit expression for the diversity distribution is a very difficult task. Fig. 4 illustrates the performance of random, Hadamard and algebraic rotations obtained by computer simulations. Four rotated lattices have been randomly selected: the 8-dimensional $Z_{8\_4\_random}$ lattice is obtained with $a = 4$ and its diversity is $L = 4$; the 16-dimensional $Z_{16\_8\_random}$ and $Z_{16\_14\_random}$. lattices correspond to $a = 4$ and $a = 16$ with a diversity order $L = 8$ and $L = 14$ respectively; we built the 32-dimensional rotation Z_{32_random} with $a = 8$ points on the unity circle but we were not able to determine the true value of its diversity order. As seen in Fig. 4, random orthogonal matrices perform as good as optimized algebraic rotations. Notice the poor performance of Hadamard matrices, especially for low dimensions.

4 Minimum MSE decoding of lattices with DFE

Equalizers are commonly used in digital communication systems to reduce the intersymbol interference (ISI) when transmitting over bandwidth limited channels [15]. When the channel impulse response is short, a maximum likelihood equalization is possible by applying the Viterbi algorithm to the channel trellis. Otherwise, the ISI reduction is done by sub-optimal but less complex equalizers based on the MSE criterion [15].

What is the relation between equalization and lattice decoding ?

A lattice Λ is a discrete set of points in the n-dimensional space $\mathbb{R}^n$ or $\mathbb{C}^n$ obtained by a linear transform of the group $\mathbb{Z}^n$, i.e. $\Lambda = M\mathbb{Z}^n$ where M is the lattice generator matrix. The effect of this matrix on $\mathbb{Z}^n$ is similar to an ISI channel : a component of a lattice point is a linear combination of all the input integers. Thus, suppressing the ISI is equivalent to decoding Λ and hence the lattice decoding can be performed with the help of an equalizer. Due to the dramatic complexity of trellis equalization (for high dimension lattices), the only possible solution is to decode the lattice with a sub-optimal MSE equalizer and decision feedback.

Fig. 5 shows the system model and the DFE with a forward matrix W and a backward matrix G. The additive white Gaussian noise b has a variance N_0 per component. The estimation of the i^{th} component is not used in the feedback equalization of the i^{th} received symbol, and so we impose the following condition

$$\forall i \in \{0, ... n-1\} \quad \mid g_{ii} \mid = 0. \tag{9}$$

We denote the transmitted vector by $x = Mz$ and the received vector by $r = x + b$. The vector $\tilde{z}$ is the input of the threshold detector and the estimated vector $\hat{z}$ is fed back to G. Let σ_z^2 denote the variance per component in the integer vector z. We assume that $\sigma_z^2 = 1$ otherwise N_0 must be replaced by N_0/σ_z^2. It is also assumed that $\mathrm{E}\left[z\hat{z}^h\right] = \rho I_n$ where ρ is a correlation factor and I_n is the identity matrix. Note that z^t is the transpose of z, z^* is the conjugate and z^h is the transpose conjugate. Practically, ρ is approximated by $\rho \approx (1 - P_e(z_i))$. Hence, $\rho = 1$ when the error rate on the integer components z_i is too small.

The DFE based on the MSE criterion minimizes the mean-square-error defined by $\mathrm{E}\left[\, ||z - \tilde{z}||^2 \,\right]$. Since the condition (9) must be taken into account, Lagrange multipliers are used and the equalizer minimizes the following quantity

$$\mathrm{E}\left(\parallel z - \tilde{z} \parallel^2 \right) - \sum_{i=0}^{n-1} \lambda_i g_{ii} \tag{10}$$

The minimization over W and G gives

$$\begin{cases} W^* = \frac{1}{N_0} D_{\rho\lambda+(1-\rho^2)} M^t V^* \quad \text{where V is defined by} \quad (\frac{1-\rho^2}{N_0} M^* M^t + I_n).V^* = I_n \\ \\ G^* = \frac{\rho}{N_0} D_{\rho\lambda+(1-\rho^2)} M^t V^* M^* + D_{\lambda-\rho} \end{cases}$$

where any D_ξ (resp. $D_{\xi\mu}$) represents the diagonal matrix $Diag(\xi_0 \dots \xi_{n-1})$, and where the vector $(\lambda_0 \dots \lambda_{n-1})$ is denoted λ.

The Lagrange multipliers λ_i are given by the constraint on G. The final expressions of W and G become

$$W = D_{\frac{1}{\rho^2 B^* + N_0}} M^h V \ and \ G = D_{\frac{\rho}{\rho^2 B^* + N_0}} M^h V M - D_{\frac{\rho B^*}{\rho^2 B^* + N_0}} \tag{11}$$

with $V^* = (v_{ij})$, $M = (M_{ij})$, $B = (B_0, \cdots, B_{n-1})$ and $B_i = \sum_{k=0}^{n-1} \sum_{l=0}^{n-1} m_{ki} v_{kl} m_{li}^*$

In the next section, the sub-optimal lattice decoder described above will be applied to two different scenarios: a dense lattice matched to the Gaussian channel and a multidimensional rotation adapted to the Rayleigh fading channel.

5 Simulation results

The fundamental gain and the kissing number of the Barnes-Wall lattice BW_{256} are $\gamma(BW_{256}) = 10.5dB$ and $\tau(BW_{256}) = 325139443200$ respectively. A finite constellation extracted from this lattice is a good signal alphabet for the Gaussian channel. The effective gain is smaller than 10.5dB due to the high kissing number. Equation (1) gives the error probability per point $P_{e_{point}}$ for an ML decoder. If the constellation binary labeling is random then $P_{e1_{bit}} = \frac{1}{2} P_{e_{point}}$, and if it is a Gray code labeling then $P_{e2_{bit}} = \frac{1}{128s} P_{e_{point}}$. $P_{e1_{bit}}$ and $P_{e2_{bit}}$ are drawn in Fig. 6 (a) for $s = 4.5$ bits per symbol (or equivalently 2.25 bits per dimension). By comparing with the 16-QAM performance we see that the practical gain of an ML decoder is 5.5dB. Fig. 6 (a) shows also the performance of the sub-optimal MSE decoder when applied to a cubic constellation extracted from BW_{256}. The MSE criterion seems far from reaching the ML criterion performance on the Gaussian channel. Indeed, the MSE decoder completely eliminates the intersymbol interference generated by the lattice structure without taking advantage of the sphere packing density.

The performance of the MSE decoder with the 256-dimensional FRT, i.e. the multidimensional algebraic rotation given by the lattice $\mathbb{Z}_{512,256}$ on the Rayleigh fading channel is shown in Fig. 6 (b). The same figure compares the FRT on the Rayleigh fading channel with the QAM on the Gaussian channel. Clearly, the Rayleigh fading channel is converted into a Gaussian channel, the fading effect is extremely reduced. The diversity distribution is good enough in large dimensions (the same argument is also valid for FFT and FHT) and compensates for the sub-optimality of the MSE criterion.

6 Conclusions

The minimum mean-square-error criterion is a way to encounter the intractability of lattice decoding in large dimensions ($n \geq 128$). The MSE decoder seems to perform poorly on the Gaussian channel. The lattice fundamental gain due to the packing density is not exploited by the MSE criterion where the optimization is done in the integer space instead of the lattice space. The MSE decoder is only capable of suppressing the lattice inherent intersymbol interference on the Gaussian channel. However, the MSE decoder performance is excellent on the Rayleigh fading channel. The high diversity orders in the lattice constellation are not completely destroyed by the sub-optimality of the decoder. When the dimension is large enough, it has been proved analytically that the ± 1 random Hadamard-like matrices exhibit an excellent diversity distribution. Computer-based simulations showed that random rotations, obtained by selecting more than two points on the unity circle, perform in the presence of fading as good as an optimized algebraic rotation $\mathbb{Z}_{n,n/2}$.

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Authors biography

Catherine Lamy was born in Vernon, France, in 1972. She received the electrical engineering degree from the École Nationale Supérieure des Télécommunications (ENST), Paris, France, in 1996, and the Diplôme d'Études Approfondies in telecommunications also in 1996.

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Fig. 1 System model.

Fig. 2 Increasing the diversity order by rotating the constellation.

Fig. 3 Diversity distribution for s=2 and dimension N (a) : N=8, (b) : N=512.

Fig. 4 Performance of Hadamard, algebraic and random rotations on the Rayleigh fading channel, 2 bits per dimension, ML decoding with CSI.

Fig. 5 Decision feedback decoding of a lattice.

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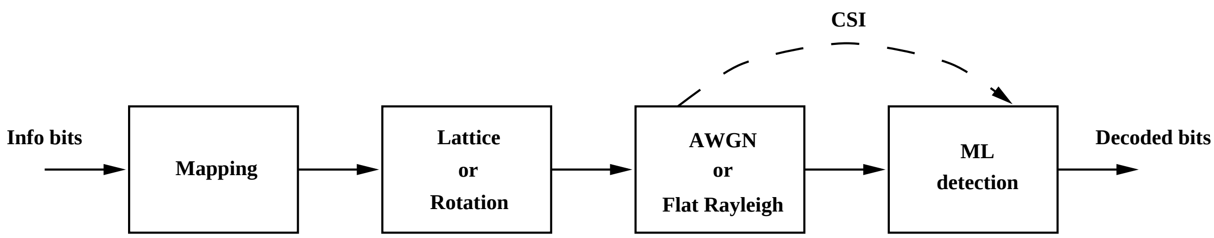

Figure 1: System model.

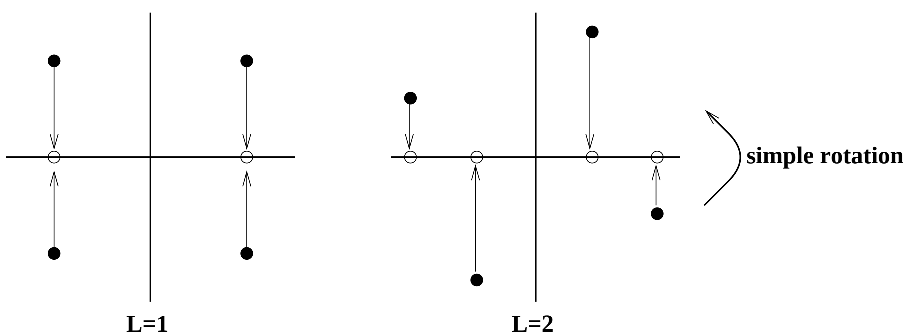

Figure 2: Increasing the diversity order by rotating the constellation.

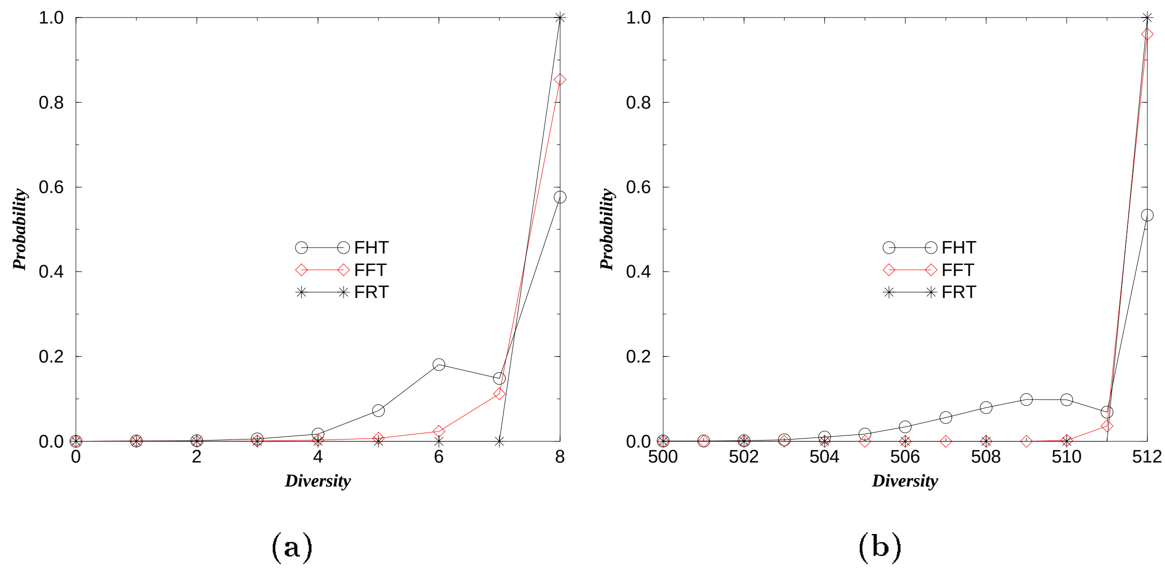

Figure 3: Diversity distribution for s=2 and dimension N (a) : N=8, (b) : N=512.

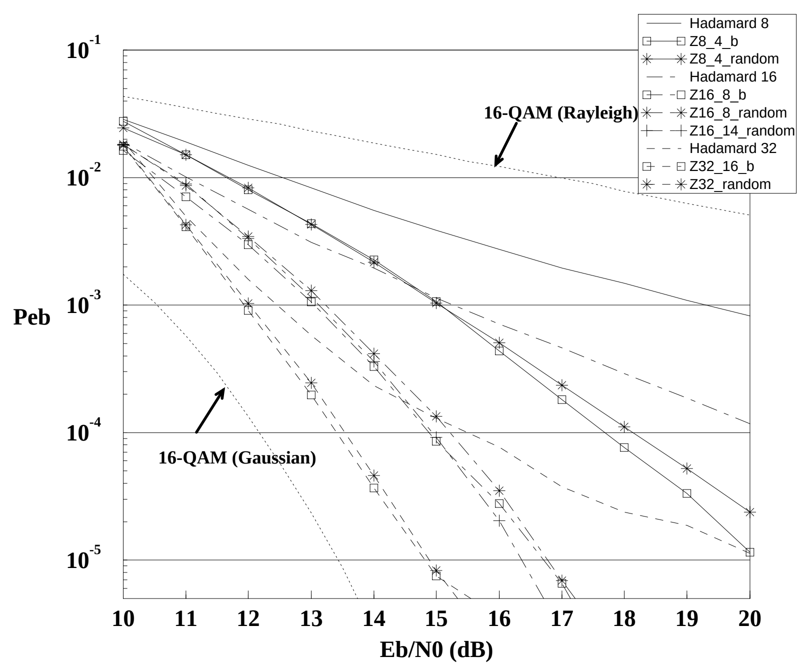

Figure 4: Performance of Hadamard, algebraic and random rotations on the Rayleigh fading channel, 2 bits per dimension, ML decoding with CSI.

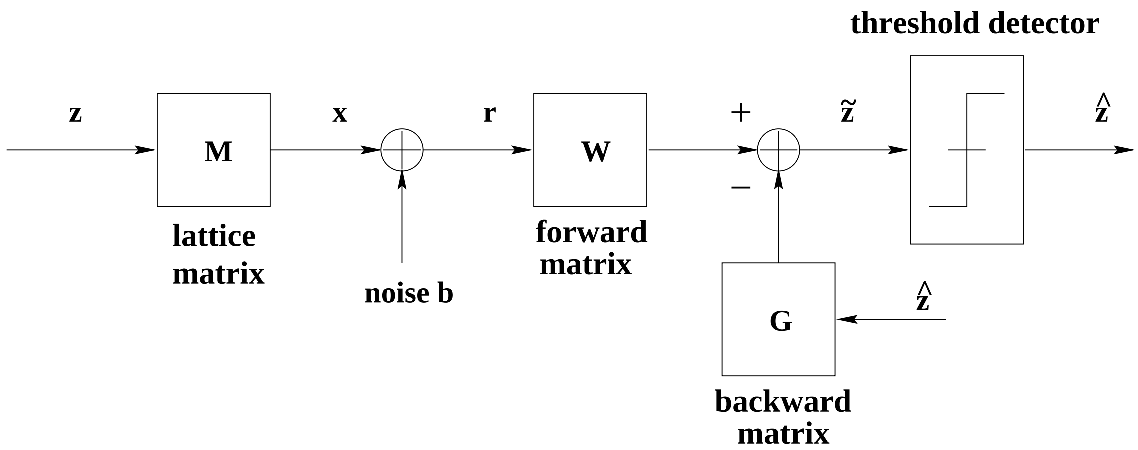

Figure 5: Decision feedback decoding of a lattice.

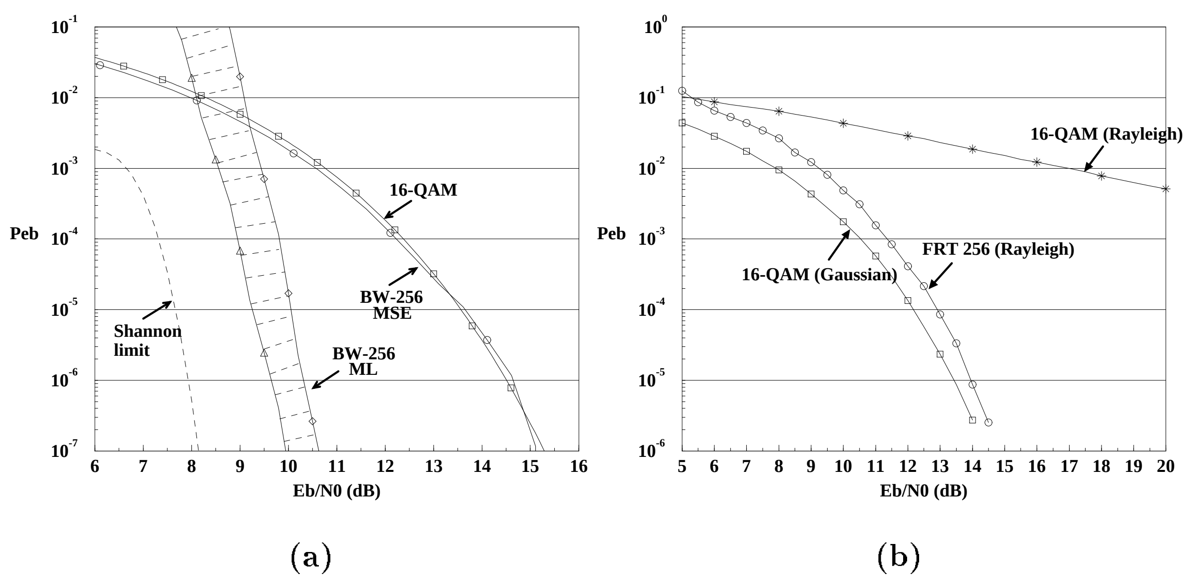

Figure 6: Binary error rate for the BW_{256} lattice, s=4.5 bits per symbol (a) and Fast Rotation Transform, dimension n=256 and s=4 bits per symbol (b).