Analysis and Optimisation of irregular LDPC codes for joint source-channel decoding

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Abstract

In this paper, we present a characterization, through its convergence analysis, and an optimisation of a joint source-channel receiver composed of a LDPC decoder and a Soft Input Soft Output (SISO) source decoder. Under Gaussian approximation, assuming the knowledge of the extrinsic mutual information transfer function (EXIT chart) of the source decoder, we derive the Mutual Information evolution equations, that semi-analytically describe the convergence of the iterative system behavior and, to complete the study, the stability condition at the convergence fixed point is derived for the joint receiver. From this analysis, a general optimisation method of the irregularity of the LDPC codes is proposed, which can be reduced to a linear programming optimisation problem. Simulation results show improved performance when compared to an AWGN optimized LDPC code.

Index Terms

LDPC codes, joint source and channel decoding (JSCD), irregularity optimisation.

I. Introduction

The interest of a joint source-channel receiver is commonly recognized, which takes advantage of both the structure and the residual redundancy of the source. Namely, [2][6] consider JSCD for serially concatenated Convolutional Codes and Variable Length Code (VLC) SISO decoders. In [5] and more recently [8], a doubly iterative system involving turbo-codes and VLC codes is investigated. In this letter is addressed the problem of the *convergence analysis and optimisation* of irregular LDPC codes to improve overall performance of a joint receiver based on a LDPC decoder and a given SISO source decoder. In Section II, the overall JSCD system is described. In Section III, the mutual information (MI) evolution is derived using a Gaussian approximation, to completely describe the convergence behavior. The mandatory stability condition (fixed point convergence stability) is also derived. Optimisation rules are then given in Section IV, together with corresponding results for an example of source code. Finally simulations results are given in Section V and conclusions and perspectives are drawn in Section VI.

II. System description and hypothesis

In the following, we consider the Binary Input-Additive White Gaussian Noise (AWGN) channel as the propagation channel. Since the LDPC decoder is itself iterative, the global joint turbo receiver could be interpreted as a

”doubly iterative” type of receiver, made of the concatenation of a LDPC decoder and a SISO source decoder. For practical considerations, we assume that the LDPC code is systematic. A decoding iteration for the global iterative receiver is composed of one LDPC decoding step and one SISO source decoder step and, as in many works, we assume initial synchronisation of the source decoder. The factor graph corresponding to the proposed system is given in Figure 1. For the purpose of the optimisation, infinite length codewords are considered. Belief propagation

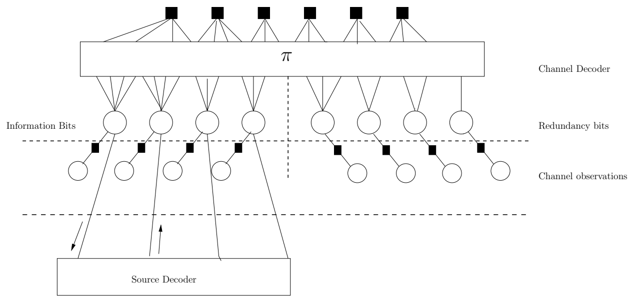

Fig. 1. Graph representation for the joint LDPC and source decoder.

(BP) is used for the LDPC decoder and Maximum A Posteriori (MAP) decoding through BCJR (equivalent to BP on a VLC factor graph) for the source decoder. The main consideration in the proposed system is that, contrarily to classical approaches, we do not insert an interleaver between the LDPC decoder and the SISO source decoder. This is based on the requirement that the source decoder must have the knowledge of the degree of the data nodes, to which it is connected, as shown in Figure 1. This crucial hypothesis is used to express the MI evolution in Section III.

III. JSCD WITH LDPC CODES: CONVERGENCE ANALYSIS

A. Notations

In our analysis, we consider the MI of the Log-Likelihood ratio (LLR) messages along the edges in the graph (see [3] for more details). As the source decoder provides an extrinsic information only for information bit, two classes of data nodes must be distinguished, namely information and redundancy data nodes (see Figure 2). At decoding iteration ℓ, we will note respectively $x_{cv}^{(\ell)}$, $x_{vc}^{I(\ell)}(i)$, $x_{vc}^{R(\ell)}(i)$, $x_{vs}^{(\ell)}(i)$ and $x_{sv}^{(\ell)}(i)$ the MI from parity check nodes to variable nodes, the MI from variable nodes with connection degree i to check nodes for information (I) data nodes, the MI from variable nodes with connection degree i to check nodes for redundancy (R) data nodes, the MI from variable nodes with connection degree i to source decoder and the MI from source decoder to variable nodes. We define $x_{vc}^{(\ell)}$ as the MI at the check node input (after the interleaver π). It is thus a mixture of $x_{vc}^{I(\ell)}(i)$ and $x_{vc}^{R(\ell)}(i)$. Using a Gaussian Approximation [4], all MI quantities can be related to the mean of LLR messages with the function $J(.)$ defined as follows [3]

$$J(m) = 1 - \frac{1}{\sqrt{4\pi m}} \int_{\mathbb{R}} \log_2 \left(1 + e^{-v}\right) \exp\left(\frac{-(v-m)^2}{4m}\right) dv.$$

Considering the LDPC code structure, are denoted by $\underline{\rho} = [\rho_2, \ldots, \rho_{t_{r_{\max}}}]^\top$, $\underline{\lambda^I} = [\lambda_2^I, \ldots, \lambda_{t_{c_{\max}}}^I]^\top$ and $\underline{\lambda^R} = [\lambda_2^R, \ldots, \lambda_{t_{c_{\max}}}^R]^\top$ respectively the proportion of edges connected to check nodes with degree $\{j, j = 2 \ldots t_{r_{max}}\}$, the proportion of edges connected to information data nodes with degree $\{i, i = 2 \ldots t_{c_{max}}\}$ and the proportion of edges connected to the redundancy data nodes with connection degree $\{r, r = 2 \ldots t_{c_{max}}\}$. $t_{r_{max}}$ (resp. $t_{c_{max}}$) is the highest available connection degree for the check nodes (resp. the data nodes). $\{\tilde{\lambda_i}^I, i = 2 \ldots t_{c_{max}}\}$ and $\{\tilde{\lambda}_r^R, r = 2 \ldots t_{c_{max}}\}$ are respectively the information data nodes proportion and the redundancy data nodes proportion. Assuming propagation on an AWGN channel, the mean of channel observations messages is $\mu_0 = 2/\sigma^2$ with σ^2 the noise variance of the channel.

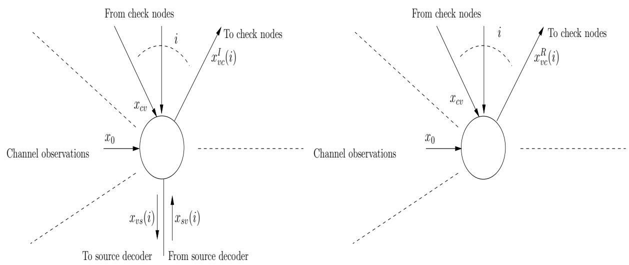

Fig. 2. Data node messages: (Left) Information data node, (Right) Redundancy data node

B. Mutual Information evolution

Assuming Gaussian Approximation for both the LDPC decoder and the SISO source decoder, we can explicit the following set of MI evolution equations:

- variable nodes messages update:

$$
\begin{aligned}
x_{vc}^{I(\ell)}(i) &= \mathrm{J}(\mu_0 + (i-1)\mathrm{J}^{-1}(x_{cv}^{(\ell-1)}) + \mathrm{J}^{-1}(x_{sv}^{(\ell-1)}(i))) \\
x_{vc}^{R(\ell)}(r) &= \mathrm{J}(\mu_0 + (r-1)\mathrm{J}^{-1}(x_{cv}^{(\ell-1)})) \\
x_{vc}^{(\ell)} &= \sum_{i=2}^{t_{c\max}} \lambda_i^I x_{vc}^{I(\ell)}(i) + \sum_{r=2}^{t_{c\max}} \lambda_r^R x_{vc}^{R(\ell)}(r)
\end{aligned}
\tag{1}
$$

- check nodes messages update:

$$
x_{cv}^{(\ell)} = 1 - \sum_{j=2}^{t_{r\max}} \rho_j \mathrm{J}((j-1)\mathrm{J}^{-1}(1 - x_{vc}^{(\ell)})) \tag{2}
$$

- LDPC decoder to source decoder messages update:

$$
x_{vs}^{(\ell)}(i) = J(\mu_0 + iJ^{-1}(x_{cv}^{(\ell)})),\ \forall i = 2, \ldots, t_{c\max} \tag{3}
$$

- source decoder messages update:

$$
x_{sv}^{(\ell)}(i) = \mathrm{T}(x_{vs}^{(\ell)}(i)),\ \forall i = 2, \ldots, t_{c\max} \tag{4}
$$

where $\mathrm{T}(.)$ is the EXIT chart function of the source decoder. Generally, $\mathrm{T}(.)$ can not be explicitly given but can be estimated using Monte Carlo simulations as done in [7] or [8] using a Gaussian Approximation. The validity of equation (4) comes from the fact that there is no interleaver between the LDPC decoder and the SISO source decoder. Equations (1), (2), (3) and (4) give the complete MI evolution

$$x_{vc}^{(\ell+1)} = F([\underline{\lambda}^I, \underline{\lambda}^R], x_{vc}^{(\ell)}, \mu_0) \tag{5}$$

for which the initial conditions are $x_{sv}^{(0)}(i) = 0, \forall i = 2, \ldots, t_{c_{\max}}$ and $x_{cv}^{(0)} = 0$. The condition $F([\underline{\lambda}^I, \underline{\lambda}^R], x, \mu_0) > x, \forall x \in [0,1]$ ensures convergence at the fixed point $x = 1$ for equation (5). This condition, called *stability condition*, gives constraints on the LDPC code profile for optimisation for a considered source function $T(.)$.

C. Stability Condition

In this section, the stability condition of the fixed point corresponding to a zero error probability is studied. The stability condition is mandatory to ensure that the joint receiver converges to a vanishing error probability [9]. This implies to study the stability of the fixed point of equation (5) at $x = 1$, which is done by resolving $F'([\underline{\lambda}^I, \underline{\lambda}^R], 1, \mu_0) < 1$. Based on results on the derivative of the function $J(.)$ from [10, pp. 1724–1725], the stability condition is expressed as follows:

Proposition 1: Under Gaussian assumption and using MI evolution, the stability condition is given by

(i) if $T(1) = 1$: $\lambda_2^R < e^{\frac{1}{2\sigma^2}} / \sum_j \rho_j (j-1)$,

(ii) if $T(1) < 1$: $\lambda_2^I e^{-\frac{M}{4}} + \lambda_2^R < e^{\frac{1}{2\sigma^2}} / \sum_{j=2}^{t_{r_{\max}}} \rho_j (j-1)$,

with $M = \mathrm{J}^{-1}(\mathrm{T}(1))$. Note that if $T(1) \to 1$, then $M \to +\infty$, so condition (ii) gives to condition (i).

As we can see, the stability condition *jointly* depends on the channel parameters and the transfert function $T(.)$.

IV. LDPC CODES OPTIMISATION FOR JSCD SYSTEMS

A. Optimisation as a linear programming problem

Looking closely to equations (1)-(4), one can see that MI evolution equation (5) is linear in the parameters $\lambda_i^I, i = 2, \ldots, t_{c_{\max}}$ and $\lambda_r^R, r = 2, \ldots, t_{c_{\max}}$. In the sequel, the following additional vector notations are used: $\underline{\lambda} = [\underline{\lambda^I}^\top, \underline{\lambda^R}^\top]^\top$, $\underline{1/t_c} = [1/2, \ldots, 1/t_{c_{\max}}]^\top$ and $\underline{1/t_r} = [1/2, \ldots, 1/t_{r_{\max}}]^\top$. Since we are interested in the optimisation of the LDPC code structure, it is proposed to maximize the rate R of the code for JSCD as done in [4]. An additional rate constraint on the proportion distribution must be considered, due to the fact that considered LDPC codes are systematic, which implies that the sum of systematic variable node proportions is equal to the rate R. After some mathematical computations for the rate constraint, the optimisation problem can finally be stated as follows:

$$\underline{\lambda}_{opt} = \max_{\underline{\lambda}} [\underline{1/t_c}^\top, \underline{1/t_r}^\top]^\top \underline{\lambda} \tag{6}$$

under the constraints:

[C$_1$] proportion constraints: $1^\top \underline{\lambda} = 1$ and $\underline{1/tc}^\top \underline{\lambda}^R = \underline{1/tr}^\top \underline{\rho}$,

[C_2] convergence constraints (Cf. eq. (5)): $\mathrm{F}\left(\underline{\lambda}, x, \mu_0\right) > x$,

[C_3] stability condition: $\lambda_2^I e^{-\frac{M}{4}} + \lambda_2^R < \lambda_2^*(\sigma^2, \underline{\rho})$.

B. *Optimisation results*

In this section, we present some obtained optimisation results. Without loss of generality (the method presented here is valid for any other SISO decoder), we consider a SISO Variable Length Code (SISO-VLC) as the source decoder. The VLC code considered as our example and the corresponding independent symbols source are taken from [2]: it consists of the codebook $\mathcal{C} = (00, 11, 010, 101, 0110)$ with associated probabilities $\mathcal{P} = (0.33, 0.30, 0.18, 0.10, 0.09)$. The associated entropy and VLC average length are respectively $H = 2.14$ and $\bar{l} = 2.46$ bits/symbols. The SISO-VLC source decoder is a Bit-level MAP VLC soft decoder introduced by [1]. The extrinsic MI transfer function is estimated through Monte Carlo simulations as in [7]. For the VLC code used, simulated EXIT charts gives the condition $T(1) \simeq 1$. As in [4], we consider concentrated degrees distribution for $\rho(x)$. We can perform the optimisation for different values of $\overline{\rho} = \rho j + (1-\rho)(j+1)$ and obtain the code with the best decoding threshold. For $t_{c_{\max}} = 30$ and $R = 1/2$, the optimal value that minimizes the threshold is $\overline{\rho} = 7.91$ and the data node connection profile obtained is given by $\lambda(x) = \lambda^I(x) + \lambda^R(x)$ with $\lambda^I(x) = 0.1130x + 0.0830x^3 + 0.1201x^4 + 0.0588x^8 + 0.1044x^9 + 0.2516x^{29}$ and $\lambda^R(x) = 0.2216x + 0.0475x^2$. The two polynomials $\lambda^I(x)$ and $\lambda^R(x)$ define the irregularity profile of the whole code. As a result of the optimisation process, low connected data node are associated with redundancy bits, whereas the highest connected are associated with information bits. It appears that the optimisation process seems to allocate the highest connected data nodes to information bits to fully benefit from the increase of information provided by the SISO-VLC decoder.

V. SIMULATION RESULTS

Considering the JSCD optimized code and the VLC code of Section IV, Figure 3 gives Bit Error Rate (BER) performance for the codeword length $N = 30000$ after 150 iterations. The residual source redundancy is given by $R_s = H/\bar{l} = 0.86992$. The overall redundancy rate is given by $R_T = R_s R = 0.43496$. At rate R_T, the Shannon limit for AWGN channel gives the theoretical information bit energy to noise ratio $E_b/N_0 = -0.0957$ dB. The BER performance is compared to the performance of the best $R = 1/2$ AWGN optimized code given by [4] with parameter $t_{c_{\max}} = 30$. In this case, $\overline{\rho} = 8.95630$. The information bits are mapped into the R data nodes with the highest connection degrees corresponding a priori to the best mapping for finite length codeword when only AWGN channel is assumed. As shown in Figure IV, the code optimized for JSCD exhibits better performance (about 0.4 dB) than the AWGN optimized one. This shows that a good LDPC code, optimized for the AWGN channel is not necessarily optimum for another type of channel.

VI. CONCLUSION

In this paper, the semi-analytical convergence analysis and a general method for the optimisation of irregular LDPC codes for joint source-channel receivers are proposed. Assuming the knowledge of the extrinsic MI transfer

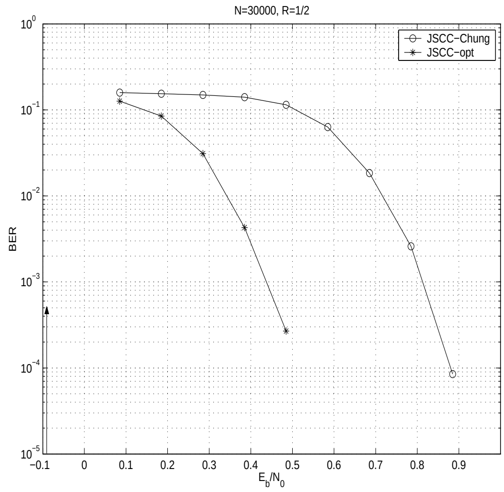

Fig. 3. BER versus E_b/N_0, $N = 30000$, $R = 1/2$ and 150 iterations

function of the source decoder, under Gaussian approximation, the joint decoder MI evolution can be written as a linear function of the LDPC parameters, as in others LDPC optimisation problems [4][10]. To complete the study, the stability condition of the global receiver was derived. The method was applied to the case of a SISO VLC decoder, but it can be generalized to any other message passing source decoder. In this paper, we focus on the optimisation of LDPC code structure for a given source decoder. The next step will be to adress the global optimisation of the joint decoder dealing with the tradeoff between the resulting redundancy after source coding and the redundancy introduced by channel coding. Further studies will also deal with the application to low complexity SISO-VLC algorithms providing less accurate extrinsic information and with the impact on performance.

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